Chapter 4
Resource Masters
Consumable Workbooks  Many of the worksheets contained in the Chapter Resource Masters are available as consumable workbooks in both English and Spanish.

<table>
<thead>
<tr>
<th></th>
<th>MHID</th>
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<tr>
<td>Study Guide and Intervention Workbook</td>
<td>0-07-878871-4</td>
<td>978-0-07-878871-0</td>
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<td>Skills Practice Workbook</td>
<td>0-07-878873-0</td>
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<td>Word Problem Practice Workbook</td>
<td>0-07-878877-3</td>
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Spanish Versions

<table>
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<tr>
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<td>0-07-878878-1</td>
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</tr>
</tbody>
</table>

Answers for Workbooks  The answers for Chapter 4 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

StudentWorks Plus™ This CD-ROM includes the entire Student Edition test along with the English workbooks listed above.

TeacherWorks Plus™ All of the materials found in this booklet are included for viewing, printing, and editing in this CD-ROM.

These masters contain a Spanish version of Chapter 4 Test Form 2A and Form 2C.
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Teacher’s Guide to Using the
Chapter 4 Resource Masters

The Chapter 4 Resource Masters includes the core materials needed for Chapter 4. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing on the TeacherWorks Plus™ CD-ROM.

Chapter Resources

**Student-Built Glossary** (pages 1–2) These masters are a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar. Give this to students before beginning Lesson 4-1. Encourage them to add these pages to their mathematics study notebooks. Remind them to complete the appropriate words as they study each lesson.

**Family Letter and Family Activity** (pages 3–6) The letter informs your students’ families of the mathematics they will be learning in this chapter. The family activity helps them to practice problems that are similar to those on the state test. A full solution for each problem is included. Spanish versions of these pages are also included. Give these to students to take home before beginning the chapter.

**Anticipation Guide** (pages 7–8) This master, presented in both English and Spanish, is a survey used before beginning the chapter to pinpoint what students may or may not know about the concepts in the chapter. Students will revisit this survey after they complete the chapter to see if their perceptions have changed.

Lesson Resources

**Lesson Reading Guide** Get Ready for the Lesson reiterates the questions from the beginning of the Student Edition lesson. Read the Lesson asks students to interpret the context of and relationships among terms in the lesson. Finally, Remember What You Learned asks students to summarize what they have learned using various representation techniques. Use as a study tool for note taking or as an informal reading assignment. It is also a helpful tool for ELL (English Language Learners).

**Study Guide and Intervention** This master provides vocabulary, key concepts, additional worked-out examples and Check Your Progress exercises to use as a reteaching activity. It can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

**Skills Practice** This master focuses more on the computational nature of the lesson. Use as an additional practice option or as homework for second-day teaching of the lesson.

**Practice** This master closely follows the types of problems found in the Exercises section of the Student Edition and includes word problems. Use as an additional practice option or as homework for second-day teaching of the lesson.
**Word Problem Practice** This master includes additional practice in solving word problems that apply the concepts of the lesson. Use as an additional practice or as homework for second-day teaching of the lesson.

**Enrichment** These activities may extend the concepts of the lesson, offer an historical or multicultural look at the concepts, or widen students’ perspectives on the mathematics they are learning. They are written for use with all levels of students.

**Graphing Calculator, Scientific Calculator, or Spreadsheet Activities** These activities present ways in which technology can be used with the concepts in some lessons of this chapter. Use as an alternative approach to some concepts or as an integral part of your lesson presentation.

**Assessment Options**

The assessment masters in the *Chapter 4 Resource Masters* offer a wide range of assessment tools for formative (monitoring) assessment and summative (final) assessment.

**Student Recording Sheet** This master corresponds with the standardized test practice at the end of the chapter.

**Pre-AP Rubric** This master provides information for teachers and students on how to assess performance on open-ended questions.

**Quizzes** Four free-response quizzes offer assessment at appropriate intervals in the chapter.

**Mid-Chapter Test** This 1-page test provides an option to assess the first half of the chapter. It parallels the timing of the Mid-Chapter Quiz in the Student Edition and includes both multiple-choice and free-response questions.

**Vocabulary Test** This test is suitable for all students. It includes a list of vocabulary words and 10 questions to assess students’ knowledge of those words. This can also be used in conjunction with one of the leveled chapter tests.

**Leveled Chapter Tests**

- **Form 1** contains multiple-choice questions and is intended for use with below grade level students.
- **Forms 2A and 2B** contain multiple-choice questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- **Forms 2C and 2D** contain free-response questions aimed at on grade level students. These tests are similar in format to offer comparable testing situations.
- **Form 3** is a free-response test for use with above grade level students.

All of the above mentioned tests include a free-response Bonus question.

**Extended-Response Test** Performance assessment tasks are suitable for all students. Sample answers and a scoring rubric are included for evaluation.

**Standardized Test Practice** These three pages are cumulative in nature. It includes three parts: multiple-choice questions with bubble-in answer format, griddable questions with answer grids, and short-answer free-response questions.

**Answers**

- The answers for the Anticipation Guide and Lesson Resources are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add this page to your math study notebook to review vocabulary at the end of the chapter.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition/Description/Example</th>
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<tbody>
<tr>
<td>bar notation</td>
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<tr>
<td>common denominator</td>
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<td></td>
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<tr>
<td>composite</td>
<td>kahm-PAH-zuht</td>
<td>number</td>
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<tr>
<td>equivalent</td>
<td>ih-KWIH-vuh-luhnt</td>
<td>fractions</td>
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<td>factor tree</td>
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<td>greatest common factor (GCF)</td>
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<td>least common denominator (LCD)</td>
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<td>least common multiple (LCM)</td>
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<tr>
<td>multiple</td>
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<tr>
<td>Vocabulary Term</td>
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<td>Definition/Description/Example</td>
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<td>simplest form</td>
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<td>terminating decimals</td>
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<tr>
<td>Venn diagram</td>
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</table>
Family Letter

Dear Parent or Guardian:

Helping students make connections between facts they learn in school and how these facts are used in the real world is essential. In our math class, we strive to make math "realistic." We encounter fractions and percents in a variety of situations, and show students how to apply these to situations they will eventually face.

In Chapter 4, Fractions, Decimals, and Percents, your child will learn about prime factorization, greatest common factors, simplifying fractions, writing fractions as decimals, writing fractions and decimals as percents, least common multiple, comparing and ordering rational numbers, and to make an organized list. In the study of this chapter, your child will complete a variety of daily classroom assignments and activities and possibly produce a chapter project.

By signing this letter and returning it with your child, you agree to encourage your child by getting involved. Enclosed is an activity you can do with your child that practices how the math we will be learning in Chapter 4 might be tested. You may also wish to log on to www.ca.gr6math.com for self-check quizzes and other study help. If you have any questions or comments, feel free to contact me at school.

Sincerely,

Signature of Parent or Guardian __________________________ Date ________
1. Use the factor tree started below to help you find the prime factorization of 64.

\[
\begin{array}{c}
64 \\
\downarrow \\
8 \quad 8 \\
\downarrow \downarrow \\
2 \quad 2
\end{array}
\]

Which shows the prime factorization of 64?

A. \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\)
B. \(2^5\)
C. \(64 \div 6\)
D. \(2 \cdot 2^3\)

**Solution**

1. Hint: Prime factorization is the expression of a number as the product of prime numbers. A prime number is a number that is only divisible by 1 and itself.

The prime factorization of 64 is shown below.

\[
\begin{array}{c}
64 \\
\downarrow \\
8 \quad 8 \\
\downarrow \downarrow \\
2 \quad 4 \quad 2 \quad 4 \\
\downarrow \downarrow \downarrow \downarrow \\
2 \quad 2 \quad 2 \quad 2
\end{array}
\]

It can be expressed as \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\) or \(2^6\).

The answer is A.

2. Twenty-five percent is represented on the grid below.

[Grid image]

Which fractions are equivalent to 25%?

A. \(\frac{1}{5}, \frac{2}{10}, \frac{3}{15}\)
B. \(\frac{1}{3}, \frac{2}{6}, \frac{3}{9}\)
C. \(\frac{1}{8}, \frac{2}{16}, \frac{3}{24}\)
D. \(\frac{1}{4}, \frac{2}{8}, \frac{3}{12}\)

**Solution**

2. Hint: As shown in the shaded region, 25% is \(\frac{25}{100}\) or \(\frac{1}{4}\), so any fraction that is equivalent to 25% should reduce to \(\frac{1}{4}\). In options A through C, a reduced fraction other than \(\frac{1}{4}\) is listed, such as \(\frac{1}{5}, \frac{1}{3}, \text{ and } \frac{1}{8}\). These options can be eliminated.

Option D contains \(\frac{1}{4}\) and two other fractions that can be reduced to \(\frac{1}{4}\), so all of the fractions listed are equivalent to 25%.

The answer is D.
Estimado padre o apoderado:

Resulta esencial hacer conexiones entre los hechos que se aprenden en la escuela y cómo usarlos en el mundo real. En nuestra clase de matemáticas, nos esforzamos por hacerla “real”. Encontramos fracciones y porcentajes en una variedad de situaciones y mostramos a los alumnos cómo aplicarlos en situaciones que enfrentarán a la larga.

En el Capítulo 4, Fracciones, decimales y porcentajes, su hijo(a) aprenderá sobre factores primos, el máximo común divisor, cómo reducir fracciones, a escribir fracciones en forma decimal, a escribir fracciones y decimales en forma de porcentajes, sobre el mínimo común múltiplo, a comparar y ordenar números racionales y a crear una lista organizada. En el estudio de este capítulo, su hijo(a) completará una variedad de tareas y actividades diarias y es posible que trabaje en un proyecto del capítulo.

Al firmar esta carta y devolverla con su hijo(a), usted se compromete a ayudarlo(a) a participar en su aprendizaje. Junto con esta carta, va incluida una actividad que puede realizar con él(ella) y la cual practica lo que podrían encontrar en las pruebas de los conceptos matemáticos que aprenderán en el Capítulo 4. Además, visiten ca.gr6math.com para ver autocontroles y otras ayudas para el estudio. Si tiene cualquier pregunta o comentario, por favor contácteme en la escuela.

Cordialmente,

Firma del padre o apoderado ___________________________ Fecha ___________
Actividad en familia
Práctica de estándares
Doblen la página a lo largo de las líneas punteadas. Resuelvan cada problema en otra hoja de papel. Luego, desdoblen la página y revisen las respuestas.

1. Usen el siguiente árbol de factores que se inició como ayuda para calcular la factorización prima de 64.

¿Cuál de los siguientes muestra la factorización prima de 64?
A $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
B $2^5$
C $64 \div 6$
D $2 \cdot 2^3$

La respuesta es A.

2. El cuadriculado siguiente representa veinticinco por ciento.

¿Qué fracciones equivalen a 25%?
A $\frac{1}{5}, \frac{2}{10}, \frac{3}{15}$
B $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}$
C $\frac{1}{8}, \frac{2}{16}, \frac{3}{24}$
D $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}$

La respuesta es D.
Anticipation Guide
Fraction, Decimals, and Percents

Before you begin Chapter 4

• Read each statement.
• Decide whether you Agree (A) or Disagree (D) with the statement.
• Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

After you complete Chapter 4

• Reread each statement and complete the last column by entering an A (Agree) or a D (Disagree).
• Did any of your opinions about the statements change from the first column?
• For those statements that you mark with a D, use a separate sheet of paper to explain why you disagree. Use examples, if possible.

<table>
<thead>
<tr>
<th>STEP 1 A, D, or NS</th>
<th>Statement</th>
<th>STEP 2 A or D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>A composite number is a number with two or more digits.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>A factor tree is used to find the prime factorization of a number.</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>The greatest common factor of two prime numbers is 1.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Two fractions are equivalent only if they have the same numerator and the same denominator.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>A fraction is in simplest form only when the greatest common factor of the numerator and denominator is 1.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>( \frac{1}{2} ) is equivalent to 1.2.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>To write a fraction as a decimal, divide the denominator into the numerator.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>The following are all ratios: 12 out of 65, 12:65, and ( \frac{12}{65} ).</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>To write ( \frac{25}{35} ) as a percent, first you must write the fraction in simplest form.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Moving a decimal point two places to the right is the same as dividing by 100.</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>To find the least common multiple of two numbers, make a list of multiples of both numbers.</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>When comparing two fractions, first rewrite the fractions with a common denominator.</td>
<td></td>
</tr>
</tbody>
</table>
# Ejercicios preparatorios

**Fracciones, decimales y porcentajes**

## PASO 1

**Antes de comenzar el Capítulo 4**

- Lee cada enunciado.
- Decide si estás de acuerdo (A) o en desacuerdo (D) con el enunciado.
- Escribe A o D en la primera columna O si no estás seguro(a) de la respuesta, escribe NS (No estás seguro(a).

<table>
<thead>
<tr>
<th>PASO 1 A, D o NS</th>
<th>Enunciado</th>
<th>PASO 2 A o D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Un número compuesto es uno que tiene dos o más dígitos.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Se usa un diagrama de árbol para encontrar la factorización prima de un número.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Dos fracciones son equivalentes sólo si tienen los mismos numeradores y denominadores.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Una fracción está en forma reducida sólo cuando el máximo común divisor del denominador y del numerador es 1.</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>$\frac{1}{2}$ equivale a 1.2.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Para escribir una fracción como un decimal, divide el denominador entre el numerador.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>Las siguientes son todas razones: 12 de 65, 12:65 y $\frac{12}{65}$.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>Para escribir $\frac{25}{35}$ como un porcentaje, primero debes escribir la fracción en forma reducida.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>Mover un punto decimal dos lugares a la derecha es lo mismo que dividir entre 100.</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>Para encontrar el mínimo común múltiplo de dos números, enumera los múltiplos de ambos números.</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>Al comparar dos fracciones, primero vuelve a expresar las fracciones con un denominador común.</td>
<td></td>
</tr>
</tbody>
</table>

## PASO 2

**Después de completar el Capítulo 4**

- Vuelve a leer cada enunciado y completa la última columna con una A o una D.
- ¿Cambió cualquiera de tus opiniones sobre los enunciados de la primera columna?
- En una hoja de papel aparte, escribe un ejemplo de por qué estás en desacuerdo con los enunciados que marcaste con una D.
Get Ready for the Lesson
Read the introduction at the top of page 181 in your textbook.
Write your answers below.
1. Using your grid paper, draw as many different rectangles as possible containing 3, 4, 5, 6, 7, 8, 9, and 10 squares.

2. Which number of squares can be drawn in only one rectangle? In more than one rectangle?

Read the Lesson
3. What is the difference between a prime and a composite number?

4. How do you know when a factor tree is complete?

5. Find the prime factorization of 28 using either method shown in Example 3.

6. How can an algebraic expression be factored?

Remember What You Learned
7. Describe in your own words how to use a factor tree to find the prime factorization of a number. Include an example as an explanation.
A whole number is **prime** if it has exactly two factors, 1 and itself. A whole number is **composite** if it is greater than one and has more than two factors. To determine the **prime factorization** of a number use a **factor tree**.

**Example 1**

Determine whether each number is **prime** or **composite**.

a. 11  

b. 24

a. The number 11 has only two factors, 1 and 11, so it is prime.

b. The number 24 has 8 factors, 1, 2, 3, 4, 6, 8, 12, and 24. So, it is composite.

**Example 2**

Determine the prime factorization of 48.

Use a factor tree.

```
    48
   /  \
  2   24
   / \
  2  12
   /   /
  2  3  4
   / /   /
 2 3 2 2
```

The prime factorization of 48 is $2 \times 2 \times 2 \times 3$ or $2^4 \times 3$.

**Exercises**

Determine whether each number is prime or composite.

1. 27  
2. 31  
3. 46  
4. 53  
5. 11  
6. 72  
7. 17  
8. 51

Determine the prime factorization of the following numbers.

9. 64  
10. 100  
11. 45  
12. 81
Determine whether each number is prime or composite.

1. 36
2. 71
3. 18
4. 27
5. 37
6. 61
7. 32
8. 21
9. 40

Find the prime factorization of each number.

10. 425
11. 82
12. 93
13. 142
14. 45
15. 56
16. 63
17. 236
18. 12
19. 110
20. 46
21. 84
Determine whether each number is prime or composite.

1. 45  2. 17  3. 21
4. 51  5. 11  6. 71
7. 3  8. 27  9. 47

Find the prime factorization of each number.

10. 88  11. 39  12. 75
13. 124  14. 165  15. 225
16. 100  17. 91  18. 27

ALGEBRA  Factor each expression.

19. 20xy  20. 18bc  21. 11pqr

22. 36g^2h^2  23. 44m^2n  24. 25z^2

Replace each • with prime factors to make a true sentence.

25. 2^2 • • 7 = 252  26. 2 • • 5^3 = 750  27. 2^3 • • 3^2 = 1,800

28. ALGEBRA  Is 2x + y prime or composite if x = 2 and y = 7?

29. ATHLETICS  The distance around an oval running track is 440 yards. Write this distance as a product of primes.
# Word Problem Practice

## Prime Factorization

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. AGE</strong></td>
<td>The average life expectancy in the United States is now 77.6 years. Round to the nearest whole number, and write it as a product of primes</td>
</tr>
<tr>
<td><strong>2. FOOTBALL</strong></td>
<td>A football team's record for the season is 12 wins and 4 losses. Write their record as a product of primes.</td>
</tr>
<tr>
<td><strong>3. BASKETBALL</strong></td>
<td>The average height of players in the NBA is 6 feet 7 inches. Write this height in inches as a product of primes.</td>
</tr>
<tr>
<td><strong>4. TRAVELING</strong></td>
<td>The distance between Washington, D.C. and Chicago, IL is about 590 miles by air. Write this distance as a product of primes.</td>
</tr>
<tr>
<td><strong>5. SCIENCE</strong></td>
<td>There are 118 elements in the Periodic Table. List all of the factors of 118. What type of number is this?</td>
</tr>
<tr>
<td><strong>6. READING</strong></td>
<td>A copy of <em>A Tale of Two Cities</em>, the classic written by Charles Dickens, has about 530 pages. Write this as a product of primes.</td>
</tr>
</tbody>
</table>
Perfect Numbers

A positive integer is *perfect* if it equals the sum of its factors that are less than the integer itself.

If the sum of the factors (excluding the integer itself) is greater than the integer, the integer is called *abundant*.

If the sum of the factors (excluding the integer itself) is less than the integer, the integer is called *deficient*.

The factors of 28 (excluding 28 itself) are 1, 2, 4, 7, and 14.

Since $1 + 2 + 4 + 7 + 14 = 28$, 28 is a perfect number.

Complete the table to classify each number as perfect, abundant, or deficient.

<table>
<thead>
<tr>
<th>Number</th>
<th>Divisors (Excluding the Number Itself)</th>
<th>Sum</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Show that each number is perfect.

6. 496

7. 8,128

8. **CHALLENGE** 33,550,336
**Spreadsheet Activity**

**Greatest Common Factor**

You can use a spreadsheet to find the greatest common factor of a pair of numbers.

### Example 1

Use a spreadsheet to find the greatest common factor of 32 and 52.

**Step 1**
Use the first column of the spreadsheet for the two numbers. Enter the numbers using the formula bar and press ENTER after each number.

**Step 2**
In cell A3, enter an equals sign followed by MOD(A1,A2). Then press ENTER.

**Step 3**
Click on the bottom right corner of cell A3 and drag it down several rows. This will return a list of numbers. The last number in the list is a 0. The greatest common factor is the number in the list just before the 0. The number before the 0 is 4.

So, the greatest common factor of 32 and 52 is 4.

### Example 2

Use a spreadsheet to find the greatest common factor of 64 and 432.

**Step 1**
Enter the three numbers in the second column.

**Step 2**
In cell B4, enter an equals sign followed by MOD(B1,B2). Then press ENTER.

**Step 3**
Click on the bottom right corner of cell B3 and drag it down several rows. This will return a list of numbers. The last number in the list is a 0. The greatest common factor is the number in the list just before the 0. The number before the 0 is 16.

So, the greatest common factor of 64 and 432 is 16.

### Find the greatest common factor of each pair of numbers.

1. 15 and 50
2. 27 and 32
3. 100 and 20
4. 211 and 15
5. 201 and 18
6. 18 and 81
7. 55 and 200
8. 33 and 121
9. 16 and 120
10. 14 and 21
11. 13 and 52
12. 90 and 1800
13. 225 and 15
14. 169 and 13
15. 80 and 64
Lesson Reading Guide

Greatest Common Factor

Get Ready for the Lesson

Read the introduction at the top of page 186 in your textbook. Write your answers below.

1. Who visited the Fashion Chat Room? the Music Chat Room?

2. Who visited both chat rooms?

Read the Lesson

3. What does a Venn diagram show?

4. How does a Venn diagram show relationships between elements?

5. You can find the GCF by using common factors or using common prime factors. What is the difference?

6. Find the prime factors of 20 and 24. What are the prime factors that are common to both numbers? What is the GCF?

7. How is the GCF of two numbers found if you know the prime factors common to the numbers?

Remember What You Learned

8. In your own words, describe what the GCF of two numbers is and explain one way to find it.
**The greatest common factor (GCF) of two or more numbers is the largest number that is a factor of each number. The GCF of prime numbers is 1.**

### Example 1
Find the GCF of 72 and 108 by listing factors.

- Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
- Factors of 108: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108
- Common factors: 1, 2, 3, 4, 6, 9, 12, 18, 36

The GCF of 72 and 108 is 36.

### Example 2
Find the GCF of 42 and 60 using prime factors.

**Method 1** Write the prime factorization.

- $60 = 2 \times 2 \times 3 \times 5$
- $42 = 2 \times 3 \times 7$

**Method 2** Divide by prime numbers.

- Divide both 42 and 60 by 2.
- Then divide the quotients by 3.

\[
\begin{align*}
7 & \quad 10 \\
3 & \quad 21 & \quad 30 \\
2 & \quad 42 & \quad 60
\end{align*}
\]

The common prime factors are 2 and 3. The GCF of 42 and 60 is $2 \times 3$, or 6.

### Exercises
Find the GCF of each set of numbers.

1. 18, 30  
2. 60, 45  
3. 24, 72  
4. 32, 48  
5. 100, 30  
6. 54, 36  
7. 3, 97, 5  
8. 4, 20, 24  
9. 36, 9, 45  

Chapter 4  17  Glencoe California Mathematics, Grade 6
## Skills Practice
### Greatest Common Factor

Find the GCF of each set of numbers.

<table>
<thead>
<tr>
<th>Set</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>14, 20</td>
</tr>
<tr>
<td>2.</td>
<td>16, 42</td>
</tr>
<tr>
<td>3.</td>
<td>8, 18</td>
</tr>
<tr>
<td>4.</td>
<td>24, 36</td>
</tr>
<tr>
<td>5.</td>
<td>72, 22</td>
</tr>
<tr>
<td>6.</td>
<td>77, 15</td>
</tr>
<tr>
<td>7.</td>
<td>32, 80</td>
</tr>
<tr>
<td>8.</td>
<td>90, 120</td>
</tr>
<tr>
<td>9.</td>
<td>45, 30</td>
</tr>
<tr>
<td>10.</td>
<td>12, 62</td>
</tr>
<tr>
<td>11.</td>
<td>15, 27</td>
</tr>
<tr>
<td>12.</td>
<td>21, 28</td>
</tr>
<tr>
<td>13.</td>
<td>12, 20, 26</td>
</tr>
<tr>
<td>14.</td>
<td>15, 20, 25</td>
</tr>
<tr>
<td>15.</td>
<td>60, 72, 36</td>
</tr>
<tr>
<td>16.</td>
<td>32, 48, 64</td>
</tr>
<tr>
<td>17.</td>
<td>36, 48, 30</td>
</tr>
<tr>
<td>18.</td>
<td>28, 56, 42</td>
</tr>
<tr>
<td>19.</td>
<td>80, 110, 90</td>
</tr>
<tr>
<td>20.</td>
<td>9, 25, 49</td>
</tr>
</tbody>
</table>

Find the GCF of each set of algebraic expressions.

<table>
<thead>
<tr>
<th>Expression Set</th>
<th>Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.</td>
<td>21ab, 14b</td>
</tr>
<tr>
<td>22.</td>
<td>20a^2, 36a</td>
</tr>
<tr>
<td>23.</td>
<td>15ab, 5b^2</td>
</tr>
<tr>
<td>24.</td>
<td>35a^2, 85ab</td>
</tr>
</tbody>
</table>

| 25. Find the GCF of | 2^3 \times 3^2 \times 5 and 2^2 \times 3 \times 5^2. |
Find the GCF of each set of numbers.

1. 16, 44
2. 15, 35
3. 24, 32
4. 27, 63
5. 20, 80
6. 18, 38
7. 14, 49
8. 66, 99
9. 9, 35
10. 6, 24, 42
11. 30, 50, 70
12. 32, 48, 96
13. 10w, 5w
14. 16xy, 24xy
15. 21ab, 35a
16. 10jk, 15k
17. 3mn, 9mn, 12mn
18. 6xy, 9x, 3y

Find two numbers whose GCF is the given number.

21. 10
22. 8
23. 14

24. SPORTS CARDS  Jason wants to organize his sports cards in packets for each type of sport. Each packet has the same number of cards. If he has 24 baseball cards, 60 hockey cards, and 48 football cards, find the greatest number of cards in each packet.

25. FORESTRY  A forest ranger needs to remove three tree trunks by cutting the trunks into equal lengths. If the lengths of the tree trunks are 6 feet, 8 feet, and 12 feet, what is the length of the longest log that can be cut?
### Word Problem Practice
#### Greatest Common Factor

1. **TABLE TENNIS**  Rebecca has 20 table tennis balls and 16 table tennis paddles. She wants to sell packages of balls and paddles bundled together. What is the greatest number of packages she can sell with no leftover balls or paddles?

2. **TUMBLING**  Mr. Nicolet wants to organize equal-sized groups of boys and girls for tumbling exercises. If there are 12 boys and 18 girls and each group is all boys or all girls, what is the largest size group he can organize?

3. **BAKE SALE**  Volunteers at a bake sale want to sell slices of banana nut bread and raisin bread packaged together. They have 63 slices of banana nut bread and 45 slices of raisin bread, and they plan to use all the bread. What is the greatest number of packages they can put together? How many slices of each type of bread are in a package?

4. **DOG TREATS**  Krista wants to give her dog a special treat. She has 81 dog bones and 54 pieces of beef jerky. If she wants to give her dog the same number of treats every day, what is the greatest number of days she can feed the dog these treats? How many of each type should she give the dog?

5. **FRUIT TREES**  Mr. Farber has 84 pear trees and 180 apple trees. He wants to plant the trees in rows of equal width. Find the most trees that can be planted in a row if each row has only one type of tree.

6. **BOARDS**  A scouting troop has three boards of lengths 14 feet, 28 feet, and 21 feet. If the boards must be cut to produce equal-sized pieces, what is the longest piece that can be cut with no waste?
Sundaram’s Sieve

This arrangement of numbers is called Sundaram’s Sieve. Like the Sieve of Eratosthenes, Sundaram’s arrangement can be used to find prime numbers.

<table>
<thead>
<tr>
<th></th>
<th>4</th>
<th>7</th>
<th>10</th>
<th>13</th>
<th>16</th>
<th>19</th>
<th>22</th>
<th>25</th>
<th>28</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>12</td>
<td>17</td>
<td>22</td>
<td>27</td>
<td>32</td>
<td>37</td>
<td>42</td>
<td>47</td>
<td>52</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>17</td>
<td>24</td>
<td>31</td>
<td>38</td>
<td>45</td>
<td>52</td>
<td>59</td>
<td>66</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>22</td>
<td>31</td>
<td>40</td>
<td>49</td>
<td>58</td>
<td>67</td>
<td>76</td>
<td>85</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>27</td>
<td>38</td>
<td>49</td>
<td>60</td>
<td>71</td>
<td>82</td>
<td>93</td>
<td>104</td>
<td>115</td>
<td></td>
</tr>
</tbody>
</table>

Here’s how to use Sundaram’s Sieve to find prime numbers. If a number, \( n \), is not in the Sieve, then \( 2n + 1 \) is a prime number. If a number, \( n \), is in the Sieve, then \( 2n + 1 \) is not a prime number.

- 32 is in the sieve. \( 2 \times 32 + 1 = 65 \) 65 is not prime.
- 35 is not in the sieve. \( 2 \times 35 + 1 = 71 \) 71 is prime.

1. Does the sieve give all primes up to 99? all the composites?

2. Sundaram’s Sieve is constructed from arithmetic sequences. Describe the pattern used to make the first row.

3. How is the first column constructed?

4. How are the second through fifth rows constructed?

5. How would you add a sixth row to the sieve?

6. Use Sundaram’s Sieve to find 5 four-digit prime numbers. You will need to add more numbers to the sieve to do this.
Example 1  LUNCH  Walnut Hills School has a deli line where students are able to select a meat sandwich, a side, and fruit. Meat choices are ham or turkey. The side choices are pretzels or chips. Fruit options are an apple or a pear. How many different combinations are possible?

Explore  You know that students can choose a sandwich, a side, and fruit. There are 2 meat choices, 2 side choices, and 2 fruit choices. You need to find all possible combinations.

Plan  Make an organized list.

Solve

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat</td>
<td>Ham</td>
<td>Ham</td>
<td>Ham</td>
<td>Ham</td>
<td>Turkey</td>
<td>Turkey</td>
<td>Turkey</td>
<td>Turkey</td>
</tr>
<tr>
<td>Side</td>
<td>Pretzel</td>
<td>Pretzel</td>
<td>Chips</td>
<td>Chips</td>
<td>Pretzel</td>
<td>Pretzel</td>
<td>Chips</td>
<td>Chips</td>
</tr>
<tr>
<td>Fruit</td>
<td>Apple</td>
<td>Pear</td>
<td>Apple</td>
<td>Pear</td>
<td>Apple</td>
<td>Pear</td>
<td>Apple</td>
<td>Pear</td>
</tr>
</tbody>
</table>

There are 8 possibilities.

Check  Draw a tree diagram to check the result.

Check

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat</td>
<td>Ham</td>
<td>Ham</td>
<td>Ham</td>
<td>Ham</td>
<td>Turkey</td>
<td>Turkey</td>
<td>Turkey</td>
<td>Turkey</td>
</tr>
<tr>
<td>Side</td>
<td>Pretzel</td>
<td>Pretzel</td>
<td>Chips</td>
<td>Chips</td>
<td>Pretzel</td>
<td>Pretzel</td>
<td>Chips</td>
<td>Chips</td>
</tr>
<tr>
<td>Fruit</td>
<td>Apple</td>
<td>Pear</td>
<td>Apple</td>
<td>Pear</td>
<td>Apple</td>
<td>Pear</td>
<td>Apple</td>
<td>Pear</td>
</tr>
</tbody>
</table>

Exercises

1. Susan has 3 shirts; red, blue, and green; 2 pants; jeans and khakis; and 3 shoes; white, black, and tan, to choose from for her school outfit. How many different outfits can she create?

2. The Motor Speedway is awarding money to the first two finishers in their annual race. If there are four cars in the race numbered 1 through 4, how many different ways can they come in first and second?
Solve by making an organized list.

1. **BAKING** Virginia and Robert have 1 dozen of each of the following types of cookies: chocolate chip, oatmeal raisin, snickerdoodles, and shortbread. If they want to divide the cookies into packages of two dozen, with one dozen of each of two types of cookie per package, how many different ways can they group the cookies?

2. **NUMBER THEORY** How many different two-digit numbers can be made using the digits 2, 9, 6, and 3?

3. **FOOD** Takanae is ordering lunch at a deli and is trying to decide what she would like on her sandwich. She has her choice of turkey, ham, or roast beef and a choice of cheddar, swiss, or muenster cheese. How many combinations of sandwich could she choose assuming that each sandwich has one type of meat and one type of cheese?

4. **TELEPHONES** How many phone numbers are possible for one area code if the first four numbers are 202-1, in that order, and the last three numbers are 1-7-8 in any order?

5. **CLOTHES** Sheila has four different shirts and two skirts with her on a business trip. How many different outfits can she create?

6. **SPORTS** Juan and Andrew are planning the schedule for a softball tournament. If there are 6 teams, how many different pairings could they make for the first tournament game?
Mixed Problem Solving

For Exercises 1 and 2, solve each problem by making an organized list.

1. VACATION Kessler, Kacy, and their parents sit in different seats in the car while driving to their grandparents for vacation. If only the parents take turns driving, how many different ways can all four people sit in the car with 2 front and 2 back seats?

2. PIZZA Everyone at the table likes pepperoni, sausage, onions, and black olives on pizza. List the different possibilities of ordering a 2-topping pizza.

Use any strategy to solve Exercises 3 and 4. Some strategies are shown below.

<table>
<thead>
<tr>
<th>PROBLEM-SOLVING STRATEGIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Use the four-step plan.</td>
</tr>
<tr>
<td>• Guess and check.</td>
</tr>
<tr>
<td>• Work backward.</td>
</tr>
<tr>
<td>• Make an organized list</td>
</tr>
</tbody>
</table>

3. NUMBER SENSE A number is increased by 12. When this sum is divided by 3, the result is the original number. What is the number?

4. COINS Three coins are tossed: a quarter, a nickel, and a dime. Complete the table showing the 8 different ways the coins could land by using H for heads and T for tails.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Nickel</th>
<th>Dime</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>

Select the Operation

For Exercises 5 and 6, select the appropriate operation(s) to solve the problem. Justify your solution(s) and solve the problem.

5. MEASUREMENT Eight furlongs is equal to one mile. If a mile is 5,280 feet, how many feet are in 5 furlongs?

6. TIME Greg works at the hardware store on weekends. He worked a total of 53 hours during the month of April. How many hours did Greg work during the last weekend in April, if he worked 14 hours, 12 hours, and 15 hours the other weekends?
# Word Problem Practice

## Problem-Solving Strategies

Solve using any method.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. NUMBER THEORY</strong></td>
<td>How many different 2-digit numbers can be made using the digits 3, 7, 8, 9, 2?</td>
</tr>
<tr>
<td><strong>2. MONEY</strong></td>
<td>Raul is shopping and stops at the drug store, grocery and the mall. He spent $6.99, $12.49, and $16.45 at each respective store, and has $11.20 in his wallet when he returns home. How much money did he have when he began shopping?</td>
</tr>
<tr>
<td><strong>3. BIRTH MONTH</strong></td>
<td>Renee is comparing her birth month to the birth month of the other girls in her class. Sherry was born two months before Renee, and Angela was born 4 months after Sherry. Corrine was born 3 months before Angela but 1 month after Allison. If Allison was born in July, in what month was Renee born?</td>
</tr>
<tr>
<td><strong>4. PATTERNS</strong></td>
<td>The following numbers form a pattern: 1, 2, 4, 8. What is the fifth number in the sequence?</td>
</tr>
<tr>
<td><strong>5. TIME</strong></td>
<td>Juanita is trying to get used to waking up earlier in the morning. She wakes up at 8:30 now, but wants to wake up at 7:15. If she wakes up five minutes earlier each morning, how many mornings will it be until she wakes up at 7:15?</td>
</tr>
<tr>
<td><strong>6. GAS MILAGE</strong></td>
<td>Julie is making a trip by car. Julie knows that her car gets 32 miles per gallon, and holds ten gallons of gasoline. If Julie has ( \frac{1}{4} ) of a tank of gas left, how many more miles can she travel before she needs to refuel?</td>
</tr>
<tr>
<td><strong>7. CLOTHES</strong></td>
<td>Carly is taking a vacation with her family and packs three pairs of shorts and four tops. How many different outfits can she make if she wears each top with each pair of shorts?</td>
</tr>
<tr>
<td><strong>8. FOOD</strong></td>
<td>Carlos is making a fruit salad and wants to use only 2 types of fruit. If he has blueberries, strawberries, grapes, oranges, and bananas on hand, how many combinations could he make?</td>
</tr>
</tbody>
</table>
Get Ready for the Lesson
Complete the Mini Lab at the top of page 192 in your textbook. Write your answers below. Show your shading.

1. Write a fraction to describe each figure: \( \frac{\text{number of shaded parts}}{\text{total number of parts}} \).

2. Based on the figures, what can you conclude about the fractions?

Read the Lesson
3. How do you find the simplest form of a fraction?

4. When you find the simplest form of a fraction, how can you check to make sure your answer is correct?

5. Use canceling to simplify the fraction \( \frac{2 \times 3 \times 7 \times 11}{3 \times 11 \times 17} \).

Remember What You Learned
6. Use a collection of rectangles like the one in the Mini Lab to show how to write \( \frac{15}{25} \) in simplest form.
Fractions that have the same value are called **equivalent fractions**. A fraction is in **simplest form** when the GCF of the numerator and denominator is 1.

**Example 1** Write \(\frac{36}{54}\) in simplest form.

First, find the GCF of the numerator and denominator.
- Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
- Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54
- The GCF of 36 and 54 is 18.

Then, divide the numerator and the denominator by the GCF.

\[
\frac{36}{54} = \frac{36 \div 18}{54 \div 18} = \frac{2}{3}
\]

So, \(\frac{36}{54}\) written in simplest form is \(\frac{2}{3}\).

**Example 2** Write \(\frac{8}{12}\) in simplest form.

Find the GCF of the numerator and the denominator.
- Factors of 8: 4 \(\cdot\) 2
- Factors of 12: 4 \(\cdot\) 3

The GCF of 8 and 12 is 4.

\[
\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}
\]

So, \(\frac{8}{12}\) written in simplest form is \(\frac{2}{3}\).

**Exercises**

Write each fraction in simplest form.

1. \(\frac{42}{72}\)

2. \(\frac{40}{64}\)

3. \(\frac{21}{35}\)

4. \(\frac{25}{100}\)

5. \(\frac{99}{132}\)

6. \(\frac{17}{85}\)
### 4-4 Skills Practice

**Simplifying Fractions**

Write each fraction in simplest form.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\frac{49}{70})</td>
<td>2.</td>
</tr>
<tr>
<td>4.</td>
<td>(\frac{14}{28})</td>
<td>5.</td>
</tr>
<tr>
<td>7.</td>
<td>(\frac{45}{75})</td>
<td>8.</td>
</tr>
<tr>
<td>10.</td>
<td>(\frac{56}{64})</td>
<td>11.</td>
</tr>
<tr>
<td>13.</td>
<td>(\frac{48}{66})</td>
<td>14.</td>
</tr>
</tbody>
</table>

Write two fractions that are equivalent to each fraction.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16.</td>
<td>(\frac{3}{4})</td>
<td>17.</td>
</tr>
<tr>
<td>19.</td>
<td>(\frac{14}{17})</td>
<td>20.</td>
</tr>
</tbody>
</table>
Write each fraction in simplest form.

1. \( \frac{12}{15} \)  
2. \( \frac{20}{45} \)  
3. \( \frac{8}{24} \)  
4. \( \frac{22}{30} \)

5. \( \frac{30}{90} \)  
6. \( \frac{29}{29} \)  
7. \( \frac{77}{88} \)  
8. \( \frac{32}{48} \)

9. \( \frac{21}{35} \)  
10. \( \frac{63}{99} \)  
11. \( \frac{18}{36} \)  
12. \( \frac{24}{30} \)

13. \( \frac{30}{75} \)  
14. \( \frac{12}{60} \)  
15. \( \frac{16}{36} \)  
16. \( \frac{42}{49} \)

17. \( \frac{55}{100} \)  
18. \( \frac{150}{180} \)  
19. \( \frac{35}{140} \)  
20. \( \frac{90}{133} \)

21. **STATES** Eight states in the United States start with the letter M. What fraction of states, in simplest form, begins with the letter M?

22. **MEASUREMENT** Fifteen inches is what fraction, in simplest form, of a yard?

23. **PERIMETER** A rectangle has length 7 centimeters and width 4 centimeters. What fraction of the perimeter, in simplest form, is the width?

24. **MONEY** Thirty-five cents is what fraction, in simplest form, of a dollar?

25. **AGE** Angie is 6 years old. Her dad is 30 years old. Angie's age is what fraction, in simplest form, of her dad's age?
4-4

Word Problem Practice

Simplifying Fractions

1. EXAM Mr. Bonilla gave an exam and 20 out of 25 students passed the exam. What fraction of the students passed the exam? Write the answer in simplest form.

2. GASOLINE Aisha filled her car’s 24-gallon gas tank. She took a short trip and used 8 gallons of gas. What fraction of the full gas tank was used on the trip? Write the answer in simplest form.

3. BICYCLES A local community college has 860 students. Of these 860 students, 220 ride bicycles. Write the number of bike riders as a fraction of the number of students at the college in simplest form.

4. PRESIDENTS Of the first 22 presidents, 8 were from New York. Write the number of presidents from New York as a fraction of the first 22 presidents in simplest form.

5. TIME Ten hours is what part of a day? Write the fraction in simplest form.

6. MEASUREMENT Eighteen inches is what part of a yard? Write the fraction in simplest form.
Parts of the World

It can be difficult to understand comparisons of different continents and their populations because the numbers are so large. You can make these comparisons easier to understand by writing them as fractions and using rounding to find an estimated ratio.

For example, the ratio of Asia’s population to North America’s population is

\[
\frac{501,500,000}{3,879,000,000} = \frac{5.015}{38.79}.
\]

If you divide both the numerator and denominator by 1,000, you get \(\frac{5.015}{38.79}\), which can be approximated as \(\frac{5}{40}\) or about \(\frac{1}{8}\).

1. Approximately what fraction of the world’s land area is found in South America?

2. Approximately what fraction of the world’s population is found in South America?

3. Approximately what fraction of Asia’s land area does the North America fill? What percentage is this?

4. Which continent is more crowded, Asia or North America? Explain. *Hint:* Use the example at the top of the page and your answer to Exercise 4

5. Which continent has the largest population per square kilometer? Explain.
Get Ready for the Lesson

Read the introduction at the top of page 196 in your textbook. Write your answers below.

1. What fraction of the buildings are between 600 and 900 feet tall?

2. Express this fraction using words and then as a decimal.

3. What fraction of the buildings are between 710 and 730 feet tall? Express this fraction using words and then as a decimal.

Read the Lesson

4. What is meant by the term place value?

5. In place value, what serves as the divider between ones and tenths?

6. What is the difference between a terminating decimal and a repeating decimal? Give an example of each.

Remember What You Learned

7. Work with a partner. Use a local newspaper, a favorite magazine, or the Internet. Find real-world situations that use fractions or decimals. Convert the fractions to decimals and the decimals to fractions. Exchange papers with your partner and correct each other's work.
Study Guide and Intervention

Fractions and Decimals

To write a decimal as a fraction, divide the numerator of the fraction by the denominator. Use a power of ten to change a decimal to a fraction.

Example 1
Write \( \frac{5}{9} \) as a decimal.

Method 1 Use pencil and paper.

\[
\begin{array}{c|c}
& 5 \\
\hline
9 & 5.000 \\
4 & 5 \\
5 & 0 \\
4 & 5 \\
5 & 0 \\
4 & 5 \\
5 & 0 \ \\
\end{array}
\]

The remainder after each step is 5.

You can use bar notation \( 0.\overline{5} \) to indicate that 5 repeats forever.

So, \( \frac{5}{9} = 0.\overline{5} \).

Method 2 Use a calculator.

\[
5 \div 9 = 0.55555556
\]

Example 2
Write 0.32 as a fraction in simplest form.

\[
0.32 = \frac{32}{100} \quad \text{The 2 is in the hundredths place.}
\]

\[
= \frac{8}{25} \quad \text{Simplify.}
\]

Exercises

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

1. \( \frac{8}{10} \)
2. \( \frac{3}{5} \)
3. \( \frac{7}{11} \)
4. \( 4\frac{7}{8} \)
5. \( \frac{13}{15} \)
6. \( 3\frac{47}{99} \)

Write each decimal as a fraction in simplest form.

7. 0.14
8. 0.3
9. 0.94
Write each repeating decimal using bar notation.

1. 0.7353535...
2. 0.424242...
3. 5.126126126...

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

4. \( \frac{3}{5} \)
5. \( \frac{19}{20} \)
6. \( 3\frac{4}{5} \)

7. \( \frac{23}{50} \)
8. \( 1\frac{5}{8} \)
9. \( \frac{19}{25} \)

10. \( 4\frac{17}{37} \)
11. \( 5\frac{3}{11} \)
12. \( \frac{17}{24} \)

13. \( 6\frac{7}{32} \)
14. \( 7\frac{9}{22} \)
15. \( 1\frac{17}{48} \)

Write each decimal as a fraction in simplest form.

16. 0.8
17. 0.52
18. 0.92

19. 0.48
20. 0.86
21. 0.76
Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

1. \( \frac{5}{8} \)  
2. \( \frac{2}{9} \)  
3. \( \frac{37}{16} \)  
4. \( \frac{3}{4} \)  
5. \( \frac{27}{50} \)  
6. \( \frac{121}{25} \)  
7. \( \frac{5}{6} \)  
8. \( \frac{1}{33} \)  
9. \( \frac{62}{11} \)  
10. \( \frac{2}{3} \)  
11. \( \frac{11}{40} \)  
12. \( \frac{13}{20} \)  
13. \( \frac{83}{5} \)  
14. \( \frac{3}{10} \)  
15. \( \frac{1}{9} \)  
16. \( \frac{3}{7} \)  
17. \( \frac{111}{24} \)  
18. \( \frac{7}{32} \)

Write each decimal as a fraction or mixed number in simplest form.

19. 0.4  
20. 0.83  
21. 3.75  
22. 2.42  
23. 0.16  
24. 0.65  

25. **KILOMETERS** One kilometer is approximately 0.62 mile. What fraction represents this length?

26. **MARATHON** Jake completed a marathon race in 3 hours and 12 minutes. Write Jake’s running time as a decimal.
1. **BOYS AND GIRLS** There were 6 girls and 18 boys in Mrs. Johnson's math class. Write the number of girls as a fraction of the number of boys. Then write the fraction as a repeating decimal.

2. **CATS** In a neighborhood of 72 families, 18 families own one or more cats. Write the number of families who own one or more cats as a fraction. Then write the fraction as a decimal.

3. **CELLULAR PHONES** In Italy, about 74 of every 100 people use cellular telephones. Write the fraction of cellular phone users in Italy. Then write the fraction as a decimal.

4. **FRUITS** Ms. Rockwell surveyed her class and found that 12 out of the 30 students chose peaches as their favorite fruit. Write the number of students who chose peaches as a fraction in simplest form. Then write the fraction as a decimal.

5. **TRAVEL** Tora took a short trip of 320 miles. He stopped to have lunch after he had driven 120 miles. Write the fraction of the trip he had completed by lunch in simplest form. Then write the fraction as a decimal.

6. **VOTING** In a recent school election, 208 of the 325 freshmen voted in their class election. Write the fraction of freshmen who voted. Then write the fraction as a decimal.
Writing Repeating Decimals as Fractions

All fractions can be written as decimals that either terminate or repeat. You have learned how to use a power of 10 to write a terminating decimal as a fraction. Below, you will study a strategy to write a repeating decimal as a fraction.

For Exercises 1–4, write each fraction as a decimal. Use bar notation if the decimal is a repeating decimal.

1. \(\frac{1}{9}\)
2. \(\frac{2}{9}\)
3. \(\frac{3}{9}\)
4. \(\frac{5}{9}\)

5. Describe the relationship between the numerator of each fraction and its decimal equivalent.

For Exercises 6–9, write each fraction as a decimal. Use bar notation if the decimal is a repeating decimal.

6. \(\frac{7}{99}\)
7. \(\frac{24}{99}\)
8. \(\frac{37}{99}\)
9. \(\frac{82}{99}\)

10. Describe the relationship between the numerator of each fraction and its decimal equivalent.

11. Use the relationship from Exercise 10 to write the decimal 0.52 as a fraction. Check your work using long division.

12. Using your observations from Exercises 5 and 11, make a prediction about the decimal equivalent of \(\frac{127}{999}\). Check to see if your prediction was correct by using long division.

13. How are the decimal equivalents of \(\frac{4}{9}\) and \(\frac{4}{99}\) different? Explain.

For Exercises 14–19, write each decimal as a fraction. Check your answers with a calculator.

14. 0.47474747...
15. 0.22
16. 0.530
17. 0.010010010...
18. 0.3266
19. 0.00328
4-5

Scientific Calculator Activity
Fractions, Decimals, and Percents

The division key $\div$ on a calculator may be used to express a fraction as a decimal. The multiplication key $\times$ may be used to express a number as a percent. Since a calculator can only show a certain number of digits, the last digit may be rounded.

**Example 1**
Express $\frac{5}{8}$ as a decimal.

Enter: $5 \div 8$ ENTER 0.625

So, $\frac{5}{8} = 0.625$.

**Example 2**
Express 0.625 as a percent.

Enter: 0.625 $\times$ 100 ENTER 62.5

So, $0.625 = 62.5\%$.

**Exercises**

Express each fraction as a percent using a calculator.

1. $\frac{3}{25}$

2. $\frac{13}{200}$

3. $1\frac{2}{5}$

4. $\frac{14}{10}$

5. $\frac{9}{250}$

6. $1\frac{9}{10}$

7. $\frac{4}{125}$

8. $\frac{23}{400}$

9. $\frac{28}{125}$

10. $\frac{45}{80}$

11. $\frac{94}{50}$

12. $12\frac{24}{125}$
Get Ready for the Lesson

Read the introduction at the top of page 202 in your textbook. Write your answers below.

1. For each method, shade a $10 \times 10$ grid that represents the number of students that chose the method.

   ![Grids representing students' choices]

2. What fraction of the students chose the Internet?

Read the Lesson

3. There is more than one way to write a ratio. Write the ratio that compares 4 to 25 in three different ways.

4. Write the ratio in Exercise 3 as a percent.

5. How does having ratios written as percents make it easier to compare amounts?

Remember What You Learned

6. Work with a partner. Explain to your partner how to convert a ratio that does not compare a number to 100 as a percent. Then have your partner explain to you how to change from a percent to a fraction in simplest form. Both of you should use examples as well as general explanations.
A ratio is a comparison of two numbers by division. When a ratio compares a number to 100, it can be written as a percent. To write a ratio or fraction as a percent, find an equivalent fraction with a denominator of 100. You can also use the meaning of percent to change percents to fractions.

**Example 1**  Write \( \frac{19}{20} \) as a percent.

\[
\frac{19}{20} \times 5 = \frac{95}{100} = 95\%
\]

Since \( 100 \div 20 = 5 \), multiply the numerator and denominator by 5.

**Example 2**  Write 92% as a fraction in simplest form.

\[
92\% = \frac{92}{100} = \frac{23}{25}
\]

Definition of percent

Simplify.

**Exercises**

Write each ratio as a percent.

1. \( \frac{14}{100} \)  
2. \( \frac{27}{100} \)  
3. 34.5 per 100

4. 18 per 100  
5. 21:100  
6. 96:100

Write each fraction as a percent.

7. \( \frac{3}{100} \)  
8. \( \frac{14}{100} \)  
9. \( \frac{2}{5} \)

10. \( \frac{1}{20} \)  
11. \( \frac{13}{25} \)  
12. \( \frac{4}{10} \)

Write each percent as a fraction in simplest form.

13. 35%  
14. 18%  
15. 75%

16. 80%  
17. 16%  
18. 15%
Write each ratio as a percent.

1. 26 out of 100
2. 5 per 100
3. 13:100

4. $\frac{39}{100}$
5. 12.5 per 100
6. 51 out of 100

Write each fraction as a percent.

7. $\frac{7}{10}$
8. $\frac{6}{50}$
9. $\frac{13}{20}$

10. $\frac{30}{50}$
11. $\frac{7}{20}$
12. $\frac{12}{20}$

13. $\frac{23}{25}$
14. $\frac{3}{10}$
15. $\frac{17}{50}$

Write each percent as a fraction in simplest form.

16. 15%
17. 85%
18. 1%

19. 70%
20. 25%
21. 19%

22. 33%
23. 22%
24. 95%
Write each ratio as a percent.

1. 56 out of 100 CDs sold
2. 75 per 100 adults
3. 89.2 out of 100 hours worked
4. 26.5:100 Calories
5. $\frac{457}{8}$ out of 100 meters
6. $33\frac{1}{3}$:100 minutes

Write each fraction as a percent.

7. $\frac{6}{10}$
8. $\frac{7}{20}$
9. $\frac{21}{25}$
10. $\frac{12}{50}$
11. $\frac{1}{2}$
12. $\frac{4}{5}$
13. $\frac{20}{90}$
14. $\frac{24}{25}$

Write each percent as a fraction in simplest form.

15. 40%
16. 35%
17. 72%
18. 44%
19. 90%
20. 17%
21. 5%
22. 26%

Replace each ● with >, <, or = to make a true sentence.

23. $\frac{1}{10}$ ● 15%
24. $\frac{3}{4}$ ● 72%
25. 85% ● $\frac{17}{20}$
26. $\frac{21}{25}$ ● 21%
27. 27% ● $\frac{27}{50}$
28. $\frac{4}{5}$ ● 60%

29. SPORTS If twenty-seven out of every 50 sports fans attend at least one professional game every year, what percent of sports fans attend at least one professional game every year?

30. WEATHER It rained 18 days during the month of April. What percent of the days during the month of April did it not rain?
### Fractions and Percents

#### 1. LUNCHES
Three out of every 10 students in Mr. Chan’s class bring their lunch to school. Write this ratio as a percent.

#### 2. COMPUTERS
In 2000, 57 out of every 100 school age children (ages 6 to 17 years) had access to a computer both at home and at school. Write this ratio as a percent.

#### 3. SALES TAX
In one town, the sales tax is 8%. Write this percent as a fraction in simplest form.

#### 4. HYGEINE
Ms. Agosto surveyed her class and found that 15 out of 30 students brushed their teeth more than twice a day. What percent of students brushed more than twice a day?

#### 5. DISCOUNT
A local retail store was having a sale and offered all their merchandise at a 25% discount. Write this percent as a fraction in simplest form.

#### 6. SPACE FLIGHT
About 64% of all individuals who have flown in space from 1961 to 2001 are from the United States. Write this percent as a fraction in simplest form.
Margarita Colmenares

Margarita Colmenares is an environmental engineer. She is a native of Los Angeles and a 1981 graduate of Stanford University. In 1989, she became the first woman president of the Society of Hispanic Professional Engineers. Ms. Colmenares was recently appointed to direct an office at the U.S. Department of Education. She has a special interest in education and has traveled extensively to talk to student groups about careers in engineering.

Environmental engineers like Colmenares use mathematics to predict the effect that our actions will have on our environment. They may also recommend ways to protect the environment. On this page, you will consider some data and recommendations concerning water usage.

Refer to the graph above.

1. Which one category accounts for more than $\frac{1}{3}$ of the water usage?

2. Estimate the fraction of a person’s daily water usage that is for bath and shower.

Use the graph above. Estimate the amount of water used in each category.

3. outside uses
4. bath and shower

5. toilet
6. laundry

7. dishwasher
8. faucets

In each situation, what percent of the water used can be saved by following the recommendation?

9. Using a water-saving shower head can save 65 liters of water out of the 130 liters normally used in a five-minute shower.

10. Turning off the water while brushing your teeth can reduce the water used from 20 liters to 2 liters.
Get Ready for the Lesson

Read the introduction at the top of page 206 in your textbook. Write your answers below.

1. Write the percent of students who read for fun as a fraction.

2. Write the fraction as a decimal.

3. Compare the decimal in Question 2 with its percent form. Identify any similarities or differences.

Read the Lesson

4. Describe each step in changing a percent to a decimal.

5. Describe each step in changing a percent to a decimal by first writing the percent as a fraction.

6. Describe how to write a percent as a decimal without writing the percent as a fraction.

Remember What You Learned

7. Work with a partner. Think of a way that will help you remember which way to move the decimal when you go from a percent to a decimal and which way to move it when you go from a decimal to a percent.
Write 42.5% as a decimal.

\[
42.5\% = \frac{42.5}{100} \quad \text{Write the percent as a fraction.}
\]

\[
= \frac{42.5 \times 10}{100 \times 10} \quad \text{Multiply by 10 to remove the decimal in the numerator.}
\]

\[
= \frac{425}{1000} \quad \text{Simplify.}
\]

\[
= 0.425 \quad \text{Write the fraction as a decimal.}
\]

Write 0.625 as a percent.

\[
0.625 = 0.625 \times 100 \quad \text{Multiply by 100.}
\]

\[
= 62.5\% \quad \text{Add the % symbol.}
\]

Write each percent as a decimal.

1. 6%
2. 28%
3. 81%
4. 84%
5. 35.5%
6. 12.5%
7. 14.2%
8. 11.1%

Write each decimal as a percent.

9. 0.47
10. 0.03
11. 0.075
12. 0.914
Write each percent as a decimal.

1. 5%
2. 20%
3. 21%
4. 83%
5. 7%
6. 56%
7. 16%
8. 45%
9. 27.3%
10. 14.9%
11. 91.5%
12. 29.3%
13. 14.4%
14. 80%
15. 7.5%
16. 10\(\frac{1}{2}\)%

Write each decimal as a percent.

17. 0.06
18. 0.13
19. 0.5
20. 0.74
21. 0.14
22. 0.92
23. 0.54
24. 0.66
25. 0.192
26. 0.295
27. 0.911
28. 0.247
29. 0.4165
30. 0.2199
31. 0.7601
32. 0.4833
Write each percent as a decimal.

1. 35%  
2. 90%  
3. 5%  
4. 1%  
5. 21.8%  
6. 64.8%  
7. 4.1%  
8. 8.5%  
9. $39\frac{21}{50}$
10. $17\frac{2}{5}$
11. $40\frac{3}{4}$
12. $88\frac{3}{5}$

Write each decimal as a percent.

13. 0.4  
14. 0.8  
15. 3.7  
16. 9.1  
17. 0.77  
18. 0.03  
19. 0.25  
20. 0.59  
21. 0.375  
22. 0.123  
23. 0.005  
24. 0.6019

Replace each $\cdot$ with $>$, $<$, or $=$ to make a true sentence.

25. 1.5 $\cdot$ 15%  
26. 0.88 $\cdot$ 8.8%  
27. 33% $\cdot$ 0.33  
28. 90% $\cdot$ 0.09  
29. 0.26 $\cdot$ 27%  
30. 65.4% $\cdot$ 0.645

ANALYZE TABLES For Exercises 31–33, use the table and the information given.

The table lists the approximate milk fat content of 5 types of milk products.

<table>
<thead>
<tr>
<th>Milk Product</th>
<th>Percent Milk Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy Cream</td>
<td>36.7%</td>
</tr>
<tr>
<td>Light Cream</td>
<td>19.2%</td>
</tr>
<tr>
<td>Whole Milk</td>
<td>3.5%</td>
</tr>
<tr>
<td>Low-Fat Milk</td>
<td>1.5%</td>
</tr>
<tr>
<td>Skim Milk</td>
<td>0.05%</td>
</tr>
</tbody>
</table>

31. Which product has the highest milk fat content?

32. Find the approximate number of grams of milk fat in a 200-gram serving of whole milk.

33. Which milk product will have approximately 15.36 grams of milk fat in an 80-gram serving?
### Word Problem Practice

#### Percents and Decimals

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. AREA</strong></td>
<td>New Mexico’s land area is about 0.03 of the total area of the United States. What percent is New Mexico’s land area of the total area of the United States?</td>
</tr>
<tr>
<td><strong>2. SCALE MODEL</strong></td>
<td>A scale model of a building is 0.25 the actual size. What percent of the actual size of the building is the model?</td>
</tr>
<tr>
<td><strong>3. NFL COACHES</strong></td>
<td>Don Shula ranks among the most successful coaches in the National Football League. In his career, he won 0.665 of his games. Write the decimal as a percent.</td>
</tr>
<tr>
<td><strong>4. SOFTBALL</strong></td>
<td>Jenny’s batting average is 0.346. Write the decimal as a percent.</td>
</tr>
<tr>
<td><strong>5. VITAMINS</strong></td>
<td>A multiple vitamin contains 450 milligrams of calcium. This is 45% of the recommended daily allowance. Write the percent as a decimal.</td>
</tr>
<tr>
<td><strong>6. BASKETBALL</strong></td>
<td>Tao makes 74% of his free throws. Write the percent as a decimal.</td>
</tr>
<tr>
<td><strong>7. SALES TAX</strong></td>
<td>The sales tax in a town is 7.25%. Write the percent as a decimal.</td>
</tr>
<tr>
<td><strong>8. FIELD TRIP</strong></td>
<td>In Ms. Silver’s English class, $20\frac{1}{4}$% of the students signed up to visit a local museum. Write the percent as a decimal.</td>
</tr>
</tbody>
</table>
African-American Scientists and Inventors

When you buy a pair of shoes, you usually have a wide variety of styles, sizes, and prices to choose from. It is the work of an African-American inventor, Jan Matzeliger (1852–1889), that makes this possible. In 1882, Matzeliger patented a lasting machine that could shape the upper portion of a shoe and attach it to the sole in a fraction of the time it took to do the job by hand. Using this machine, shoe manufacturers were able to increase production and reduce prices dramatically.

African Americans have made many significant contributions to mathematics, science, and invention. By solving the percent problems and matching the problem and the correct solution, you will learn more about just a few of them.

Solutions
A. 20 Benjamin Banneker
B. 21 Majorie Lee Browne
C. 18 Lewis Latimer
D. 17.5 Jane Cooke Wright

1. 35% of 50 is what number?
This physician researched and tested chemotherapy as a method of treating cancer. In 1952, she became head of the Cancer Research Foundation at Harlem Hospital.

2. What percent of 75 is 15?
This mathematician was part of the team of surveyors who created the street plan for Washington, D.C. in the late eighteenth century.

3. 4.5% of 400 is what number?
In 1876, this engineer drew up the plans that accompanied Alexander Graham Bell's application for a patent on the telephone.

4. 120% of what number is 25.2?
In 1949, she became one of the first two African-American women to earn a doctorate in mathematics. She was head of the mathematics department at North Carolina Central University from 1951 to 1970.
Lesson Reading Guide

Least Common Multiple

Get Ready for the Lesson

Complete the Mini Lab at the top of page 211 in your textbook. Write your answers below.

1. Add a second floor to each building. Record the total number of cubes used in a table like the one shown below.

<table>
<thead>
<tr>
<th>Number of Floors</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cubes in Building 1</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Cubes in Building 2</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Add floors until each building has five floors.

3. Describe two buildings that have the same number of cubes.

4. If you keep adding floors, will the two buildings have the same number of cubes again? Explain.

Read the Lesson

5. What is a least common multiple of two or more numbers?

6. Describe, in your own words, the first method used to find the LCM in Example 1 at the bottom of page 259.

Remember What You Learned

7. Explain how to find the LCM of two or more numbers when you know the prime factorization of each number. Give an example.
# Least Common Multiple

A multiple of a number is the product of that number and any whole number. The least nonzero multiple of two or more numbers is the **least common multiple** (LCM) of the numbers.

## Example 1

Find the LCM of 15 and 20 by listing multiples.

List the multiples.
- Multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120, ...
- Multiples of 20: 20, 40, 60, 80, 100, 120, 140, ...

Notice that 60, 120, ..., are common multiples. So, the LCM of 15 and 20 is 60.

## Example 2

Find the LCM of 8 and 12 using prime factors.

Write the prime factorization.
- $8 = 2 \times 2 \times 2 = 2^3$
- $12 = 2 \times 2 \times 3 = 2^2 \times 3$

The prime factors of 8 and 12 are 2 and 3. Multiply the greatest power of both 2 and 3.

The LCM of 8 and 12 is $2^3 \times 3$, or 24.

## Exercises

Find the LCM of each set of numbers.

1. 4, 6
2. 6, 9
3. 5, 9
4. 8, 10
5. 12, 15
6. 15, 21
7. 4, 15
8. 8, 20
9. 8, 16
10. 6, 14
11. 12, 20
12. 9, 12
13. 14, 21
14. 6, 15
15. 4, 6, 8
16. 3, 5, 6
Find the LCM of each set of numbers.

1. 2, 8

2. 6, 10

3. 10, 11

4. 10, 12

5. 9, 18

6. 4, 22

7. 12, 30

8. 4, 13

9. 25, 30

10. 250, 30

11. 200, 18

12. 70, 90

13. 18, 54

14. 30, 65

15. 180, 252

16. 20, 55

17. 21, 60

18. 3, 5, 10

19. 3, 4, 13

20. 4, 10, 12

21. 6, 15, 20

22. 48, 16, 3

23. 66, 55, 44

24. 29, 58, 4
Find the LCM of each set of numbers.

1. 8, 12
2. 10, 25
3. 12, 18
4. 20, 30
5. 8, 9
6. 15, 35
7. 3, 5, 7
8. 4, 10, 12
9. 9, 12, 15
10. 5, 15, 20
11. 14, 21, 42
12. 15, 18, 30
13. 2 feet, 1 yard
14. 6¢, 18¢, 24¢
15. 40 seconds, 1 minute

Write two numbers whose LCM is the given number.

16. 24
17. 63
18. 50

19. SECURITY  In a large industrial complex, three security teams work different types of security checks. The first team makes a complete round in 3 hours, the second team makes a complete round in 2 hours, while the third team makes a complete round in 4 hours. If all three teams start security checks at 7 A.M., when will be the next time all three teams finish a security check at the same time?

20. COOKIES  A recipe for large oatmeal cookies will make 15 cookies. A recipe for chocolate chip cookies will make 2 dozen cookies. If you want to have the same number of each type of cookie, what is the least number of each that you will need to make using complete recipes?

21. ICE SKATING  Three friends ice skate at different speeds. Parcel skates one lap in 45 seconds. It takes Hansel \(1 + \frac{1}{2}\) minutes to skate one lap and Forrest takes only 30 seconds to skate a lap. If they started out together, in how many minutes will they meet next?
1. **PROMOTION** In a promotion for a local delicatessen, every eighth customer will get a free sandwich and every sixth customer will get a free drink. Which customer will be first to get both a free sandwich and a free drink?

2. **WORK** Alano and Abey both work at night. Alano has every fourth night off and Abey has every sixth night off. If they are both off tonight, how long will it be before they are both off again?

3. **RADIO** A radio station is giving away a discount coupon to every twelfth caller and a free concert ticket to every twentieth caller. Which caller will be first to win both the coupon and the ticket?

4. **MUSIC** Faith spent the same amount of money on cassette tapes as she did on CDs. If tapes cost $12 and CDs cost $16, what is the least amount of money she could have spent on each?

5. **BIKES** Three bike riders ride around a circular path. The first rider circles the path in 12 minutes, the second in 18 minutes, and the third in 24 minutes. If they all start at the same place, at the same time, and go in the same direction, after how many minutes will they meet at the starting point?

6. **PAPER GOODS** At a party store, paper cups come in packages of 15, paper plates come in packages of 30, and napkins come in packages of 20. In order to have the same number of cups, plates, and napkins, what is the least number of each that must be purchased?
Periodic Cicadas

Cicadas, also commonly known as locusts, are insects that inhabit much of the eastern United States. Some cicadas are called **periodic** because they have life cycles that span periods of several years. The *Magicicada* is a kind of cicada that has an unusually long life cycle of 13 or 17 years. These 13-year and 17-year cicadas spend much of their lives living underground. After 12 or 16 years, the cicadas start to burrow upward. All at once they emerge from the ground, taking flight and eating most of the leaves on nearby plants.

While scientists do not know for sure why the *Magicicada* life cycles last for 13 or 17 years, they do have several theories. One theory is that this life cycle pattern makes it easier for the cicadas to find food. Another theory is that the pattern helps the cicada avoid predators.

**For Exercises 1–4, refer to the following information.**

Suppose there are many kinds of cicadas. For each of the pair below, find out how many years would pass before each would again emerge at the same time.

1. 11-year and 15-year cicadas

2. 12-year and 16-year cicadas

3. 14-year and 18-year cicadas

4. 13-year and 17-year cicadas

5. In what year will the 13-year and 17-year cicadas that emerged in the summer of 1998 once again emerge at the same time? Explain.

**For Exercises 6–8, refer to the following information.**

Suppose the cicada has three predators that have life cycles of 2, 3, and 5 years. The population of cicadas was nearly wiped out one summer because all three predators emerged when the cicadas emerged.

6. After how many years will the three predators emerge together again?

7. For each of the cicada populations in Exercises 1-4, find the number of years before the cicada population would again emerge at the same time as all three predators. Record your answers in the table below.

<table>
<thead>
<tr>
<th>Cicada Type</th>
<th>11-year</th>
<th>12-year</th>
<th>13-year</th>
<th>14-year</th>
<th>15-year</th>
<th>16-year</th>
<th>17-year</th>
<th>18-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Which two cicada populations have the best chance of survival? Explain.
Use your calculator to compare fractions.

**Method A**
Express the fractions as decimals, and then compare.

*Example 1:* Which is greater \( \frac{9}{13} \) or \( \frac{12}{20} \)?

\[ \frac{9}{13} \approx 0.6923 \]
\[ \frac{12}{20} = 0.6 \]

Since \( 0.69 > 0.6 \), \( \frac{9}{13} > \frac{12}{20} \).

**Method B**
Use the calculator’s comparison feature with < and >. If the result is 0, then the statement is not true; if the result is 1, then the statement is true.

\[ \frac{9}{13} \text{ < } \frac{12}{20} \]

The result is 0.

\[ \frac{9}{13} \text{ > } \frac{12}{20} \]

The result is 1. So, \( \frac{9}{13} > \frac{12}{20} \).

**Exercises**

Replace the \( \bullet \) with <, >, or =.

1. \( \frac{3}{4} \bullet \frac{7}{9} \)
2. \( \frac{2}{5} \bullet \frac{3}{8} \)
3. \( \frac{16}{20} \bullet \frac{2}{3} \)
4. \( \frac{5}{6} \bullet \frac{8}{9} \)
5. \( \frac{8}{3} \bullet \frac{12}{5} \)
6. \( \frac{2}{5} \bullet \frac{5}{8} \)
7. \( \frac{1}{12} \bullet \frac{3}{34} \)
8. \( \frac{18}{17} \bullet \frac{17}{15} \)
Lesson Reading Guide
Comparing and Ordering Rational Numbers

Get Ready for the Lesson

Complete the Mini Lab at the top of page 215 in your textbook. Write your answers below.

1. \(\frac{7}{8}, \frac{3}{8}\)

2. \(\frac{5}{8}, \frac{11}{8}\)

3. \(\frac{13}{8}, \frac{3}{8}\)

4. \(\frac{-7}{8}, \frac{-5}{8}\)

5. \(\frac{1}{2}, \frac{3}{4}\)

6. \(1\frac{1}{4}, \frac{1}{1}\)

7. MAKE A CONJECTURE Which number is less: \(\frac{-4}{7}\) or \(\frac{-6}{7}\)? Use a number line to explain your reasoning.

Read the Lesson

8. What are two ways in which you can compare fractions?

9. Complete the table of common fraction-decimal-percent equivalents.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{5})</td>
<td>(0.2)</td>
<td>(20%)</td>
</tr>
<tr>
<td>(\frac{7}{10})</td>
<td>(0.7)</td>
<td>(70%)</td>
</tr>
</tbody>
</table>

10. How are the following sets of numbers related: whole numbers, rational numbers, integers?

Remember What You Learned

11. In this lesson you learned about the LCD. What do each of the following abbreviations stand for: LCD, LCM, and GCF? How are the LCD and LCM related?
To compare fractions, rewrite them so they have the same denominator. The least common denominator (LCD) of two fractions is the LCM of their denominators.

Another way to compare fractions is to express them as decimals. Then compare the decimals.

**Example 1** Which fraction is greater, $\frac{3}{4}$ or $\frac{4}{5}$?

**Method 1** Rename using the LCD.

\[
\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}
\]

\[
\frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20}
\]

The LCD is 20.

Because the denominators are the same, compare numerators.

Since $\frac{16}{20} > \frac{15}{20}$, then $\frac{4}{5} > \frac{3}{4}$.

**Method 2** Write each fraction as a decimal. Then compare decimals.

\[
\frac{3}{4} = 0.75
\]

\[
\frac{4}{5} = 0.8
\]

Since 0.8 > 0.75, then $\frac{4}{5} > \frac{3}{4}$.

**Exercises**

Find the LCD of each pair of fractions.

1. $\frac{1}{2}, \frac{1}{8}$
2. $\frac{1}{3}, \frac{3}{4}$
3. $\frac{3}{4}, \frac{7}{10}$

Replace each $\bullet$ with $<$, $>$, or $=$ to make a true sentence.

4. $\frac{1}{2} \bullet \frac{4}{9}$
5. $\frac{4}{5} \bullet \frac{8}{10}$

6. $\frac{3}{4} \bullet \frac{7}{8}$
7. $\frac{1}{2} \bullet \frac{5}{9}$

8. $\frac{9}{14} \bullet \frac{10}{17}$
9. $\frac{5}{7} \bullet \frac{6}{11}$

10. $\frac{8}{17} \bullet \frac{1}{2}$
11. $\frac{9}{10} \bullet \frac{17}{19}$
Find the LCD of each pair of fractions.

1. \( \frac{4}{7}, \frac{3}{5} \)  
2. \( \frac{5}{12}, \frac{7}{24} \)  
3. \( \frac{6}{28}, \frac{3}{7} \)

4. \( \frac{7}{15}, \frac{1}{4} \)  
5. \( \frac{7}{11}, \frac{3}{5} \)  
6. \( \frac{5}{17}, \frac{7}{8} \)

7. \( \frac{5}{12}, \frac{7}{10} \)  
8. \( \frac{15}{16}, \frac{1}{4} \)  
9. \( \frac{5}{8}, \frac{3}{5} \)

Replace each \( \bullet \) with <, >, or = to make a true sentence.

10. \( \frac{3}{10} \bullet \frac{2}{9} \)  
11. \( \frac{3}{7} \bullet \frac{5}{11} \)  
12. \( \frac{9}{12} \bullet \frac{3}{4} \)

13. \( \frac{12}{13} \bullet \frac{14}{15} \)  
14. \( \frac{4}{5} \bullet \frac{5}{4} \)  
15. \( \frac{17}{30} \bullet \frac{13}{20} \)

16. \( \frac{35}{60} \bullet \frac{49}{84} \)  
17. \( \frac{3}{4} \bullet \frac{7}{20} \)  
18. \( \frac{2}{3} \bullet \frac{9}{7} \)

Order each set of ratios from least to greatest.

19. 0.48, 0.46, \( \frac{9}{20} \)  
20. 0.99, 0.89, \( \frac{7}{8} \)  
21. \( \frac{1}{4}, 0.2, 0.4 \)

Determine whether each number is rational. Write yes or no.

22. 2.323323332...  
23. \( \frac{7}{19} \)  
24. 4.3
Practice
Comparing and Ordering Rational Numbers

Replace each \( \bullet \) with >, <, or = to make a true sentence.

1. \( \frac{5}{6} \bullet \frac{1}{3} \)
2. \( \frac{4}{5} \bullet \frac{9}{10} \)
3. \( \frac{6}{9} \bullet \frac{4}{6} \)
4. \( \frac{2}{7} \bullet \frac{1}{8} \)
5. \( \frac{15}{21} \bullet \frac{12}{18} \)
6. \( \frac{24}{32} \bullet \frac{36}{48} \)
7. \( \frac{8}{11} \bullet \frac{10}{13} \)
8. \( \frac{14}{15} \bullet \frac{19}{20} \)
9. \( \frac{4}{5} \bullet \frac{4}{10} \)
10. \( \frac{7}{9} \bullet \frac{7}{3} \)
11. \( \frac{17}{20} \bullet \frac{8}{10} \)
12. \( \frac{3}{2} \bullet \frac{9}{6} \)
13. 50% \( \bullet \) 8 out of 10
14. 0.65 \( \bullet \) 65 out of 100
15. 4 out of 5 \( \bullet \) 75%
16. 1 out of 3 \( \bullet \) 1.3
17. \( \frac{2}{3} \) mile \( \bullet \) \( \frac{2}{5} \) mile
18. \( \frac{7}{10} \) gram \( \bullet \) 0.72 gram

Determine whether each number is rational. Write yes or no. Explain your reasoning.

19. \( \frac{8}{21} \)
20. 0.505050505\ldots
21. 1.142857

Order each set of numbers from least to greatest.

22. 63%, \( \frac{2}{3} \), 0.65
23. \( \frac{7}{8} \), 0.98, 98.5%,
24. 0.2, 2%, \( \frac{1}{12} \)

25. **BASEBALL** The pitchers for the home team had 12 strikeouts for 32 batters, while the pitchers for the visiting team had 15 strikeouts for 35 batters. Which pitching team had a greater fraction of strikeouts?

26. **TRANSPORTION** To get to school, 38% of the students ride in the family vehicle, 5 out of 12 students ride on the school bus, and 0.12 of the students ride a bike. Order the types of transportation students use to get to school from least to greatest.
1. **RAIN** The amount of rainfall was measured after a recent storm. The north side of town received $\frac{7}{8}$ inch of rain, and the south side received $\frac{13}{15}$ inch of rain. Which side of town received more rain from the storm?

2. **MOVIES** Because he sees movies at his local theater so often, Delmar is being offered a discount. He can have either $\frac{1}{3}$ off his next ticket or 30% off his next ticket. Which discount should Delmar choose? Explain.

3. **TRACK** Willie runs the 110-meter hurdles in $17\frac{3}{5}$ seconds, and Anier runs it in $17\frac{6}{11}$ seconds. Which runner is faster?

4. **FARMING** Cassie successfully harvested $\frac{7}{12}$ of her crop, and Robert successfully harvested 58% of his crop. Which person successfully harvested the larger portion of his or her crop?

5. **TRANSPORTATION** My-Lien has enough room in her truck to move 3.385 tons of gravel. Her father has asked her to move $3\frac{5}{16}$ tons. Will My-Lien be able to move all of the gravel in only one trip? Explain.

6. **WOOD WORKING** Kishi has a bolt that is $\frac{5}{8}$ inch wide, and she drilled a hole 0.6 inch wide. Is the hole large enough to fit the bolt? Explain.

7. **PIZZA** In a recent pizza-eating contest, Alfonso ate $1\frac{3}{8}$ pizzas, Della ate $1\frac{3}{10}$ pizzas, and Delsin ate $1\frac{4}{9}$ pizzas. Which person won the contest?

8. **STUDYING** For a recent algebra exam, Pat studied $1\frac{8}{15}$ hours, Toni studied $1\frac{11}{20}$ hours, and Morgan studied $1\frac{9}{16}$ hours. List the students in order by who studied the most.
Intersection and Union of Sets

The darker shaded areas in the Venn diagrams show the union and intersection of sets $A$ and $B$.

For example, if $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then their union and intersection are written as:

Union: $A \cup B = \{1, 2, 3, 4, 5, 6\}$  
Intersection: $A \cap B = \{3, 4\}$

Draw a Venn diagram for sets $A$ and $B$. Then write the numbers included in $A \cup B$ and $A \cap B$. In Exercises 2 and 4, record the numbers as decimals.

1. $A = \{\text{integers between 0 and 7}\}$  
   $B = \{\text{factors of 12}\}$

2. $A = \{\text{one-place decimals between 0 and 0.5}\}$  
   $B = \{\text{fractions with 1, 2, 3, or 4 as numerator and 5 as a denominator}\}$

3. $A = \{\text{perfect squares between 0 and 30}\}$  
   $B = \{\text{odd whole numbers less than 10}\}$

4. $A = \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$  
   $B = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$
Student Recording Sheet

Use this recording sheet with pages 274–275 of the Student Edition.

Read each question. Then fill in the correct answer.

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  
9.  
10.  
11.  
12.  

Record your answers for Question 13 on the back of this paper.
Rubric for Scoring Pre-AP

(Use to score the Pre-AP question on page 227 of the Student Edition.)

General Scoring Guidelines

- If a student gives only a correct numerical answer to a problem but does not show how he or she arrived at the answer, the student will be awarded only 1 credit. All extended response questions require the student to show work.

- A fully correct answer for a multiple-part question requires correct responses for all parts of the question. For example, if a question has three parts, the correct response to one or two parts of the question that required work to be shown is not considered a fully correct response.

- Students who use trial and error to solve a problem must show their method. Merely showing that the answer checks or is correct is not considered a complete response for full credit.

Exercise 13 Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>The model of each ratio is drawn accurately. The classroom with the greatest number of students volunteering is determined to be Room A, and the classroom with the greatest fraction of students volunteering is determined to be Room B. The ratios are correctly written as $\frac{7}{24} = 0.2916 = 29.2%, \frac{3}{8} = 0.375 = 37.5%, \text{ and } \frac{6}{18} = \frac{1}{3} = 0.3 = 33.3%$.</td>
</tr>
<tr>
<td>3</td>
<td>The classrooms are correctly identified and the ratios are correctly written in other forms, but the models are not accurately drawn. OR The models are accurate and the classrooms are correctly identified, but there is a computational error in changing the ratios to other forms.</td>
</tr>
<tr>
<td>2</td>
<td>One of the ratios is not accurately modeled and is incorrectly written to other forms, but the other two ratios are correct. The classrooms are correctly identified. OR The classrooms are correctly identified and the ratios are correctly changed, but the models are incorrect. OR The ratios are correctly modeled and changed, but the classrooms are incorrectly identified.</td>
</tr>
<tr>
<td>1</td>
<td>Only the models or changes to other forms is correct. OR Only one ratio is modeled and changed to other forms correctly.</td>
</tr>
<tr>
<td>0</td>
<td>Response is completely incorrect.</td>
</tr>
</tbody>
</table>
Chapter 4 Quiz 1
(Lessons 4-1 through 4-3)

Find the prime factorization of each number.

1. 28
2. 44
3. 315
4. What is the prime factorization of 500?

Factor each expression.

5. 24ab
6. 12xy^2

Find the GCF of each set of numbers.

7. 30, 105
8. 4, 6, 14

Find the GCF of each set of algebraic expressions.

9. 18w, 63w^2
10. 10r^3, 55r

Chapter 4 Quiz 2
(Lessons 4-4 and 4-5)

1. Write 0.64 as a fraction in simplest form.

Write each fraction in simplest form.

2. \(\frac{12}{64}\)
3. \(\frac{30}{36}\)

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

4. \(2\frac{5}{8}\)
5. \(\frac{7}{9}\)
Chapter 4 Quiz 3
(Lessons 4-6 and 4-7)

1. Write 66% as a fraction in simplest form.
2. Write \(\frac{7}{20}\) as a percent.

Write each ratio as a percent.
3. 38:100
4. 17 out of 100 feet

Write each percent as a decimal.
5. 84%
6. 6%
7. 25\(\frac{1}{2}\)%

Write each decimal as a percent.
8. 0.28
9. 0.299
10. 0.04

MULTIPLE CHOICE
1. Find the number that is not rational.
   A. 7.\(\overline{3}\)
   B. 7.03003003...
   C. \(\frac{5}{2}\)
   D. –8.4

Find the LCM of each set of numbers.
2. 18, 24
3. 25, 30, and 36

Replace each \(\bullet\) with <, >, or = to make a true sentence.
4. \(\frac{24}{30}\) \(\bullet\) \(\frac{45}{50}\)
5. \(\frac{6}{17}\) \(\bullet\) \(\frac{18}{51}\)
Chapter 4 Mid-Chapter Test
(Lessons 4-1 through 4-5)

Write the letter for the correct answer in the blank at the right of each question.

1. Which number is composite?
   A. 37  B. 35  C. 31  D. 3
   1. ____

2. Find the prime factorization of 280.
   F. $2^2 \times 5 \times 7$  G. $2 \times 4 \times 5 \times 7$  H. $2^3 \times 35$  J. $2^3 \times 5 \times 7$
   2. ____

3. Find the GCF of 28 and 49.
   A. 2  B. 7  C. 196  D. 1,372
   3. ____

4. Find the GCF of $27ab$ and $81b^2$.
   F. $9b$  G. $27b$  H. $81b$  J. 27
   4. ____

5. Which of the following fractions is equivalent to $\frac{3}{7}$?
   A. $\frac{6}{15}$  B. $\frac{9}{21}$  C. $\frac{15}{28}$  D. $\frac{7}{3}$
   5. ____

6. Write $6\frac{1}{11}$ as a decimal using bar notation.
   F. 0.609  G. 6.09  H. 6.90  J. 60.09
   6. ____

7. Express 0.44 as a fraction.
   A. $\frac{44}{10}$  B. $\frac{411}{25}$  C. $\frac{11}{25}$  D. $\frac{22}{100}$
   7. ____

8. Is the value of $5e - 2f$ prime or composite if $e = 2$ and $f = 3$?
   8. ____________

9. Factor $100m^2n$.
   9. ____________

10. Find the GCF of $30w$ and $75w^2$.
    10. ____________

11. Find the GCF of 20, 48, and 64.
    11. ____________

12. Write $\frac{32}{48}$ in simplest form.
    12. ____________

13. **TIME** 45 minutes is what part of one hour? Write the result as a fraction in simplest form.
    13. ____________

14. Write 2.124124124... using bar notation.
    14. ____________

15. Write $7\frac{7}{8}$ as a decimal.
    15. ____________
Choose the correct term or number to complete each sentence.

1. Every composite number can be written as the product of prime numbers, which is its (prime factorization, multiple), in exactly one way.

2. Fractions like \( \frac{3}{9} \) and \( \frac{6}{18} \) are called (least common multiples, equivalent fractions) because they have the same value.

3. A (terminating decimal, repeating decimal) is a decimal that ends when it reaches a remainder of zero.

4. The (least common denominator, greatest common factor) is the greatest of the factors that are common to two or more numbers.

5. The product of a number and any whole number is called a (prime number, multiple).

6. (Rational numbers, Greatest common factors) are numbers that can be written as fractions.

7. A (prime factorization, ratio) is a comparison of two numbers by division.

8. A whole number greater than 1 that has more than two factors is called a (composite number, prime number).

9. A (Venn diagram, factor tree) uses circles to show how elements among sets of numbers are related.

10. A percent is a ratio that compares a number to (0, 100).

Define each term in your own words.

11. simplest form

12. percent
Write the letter for the correct answer in the blank at the right of each question.

1. Which number is prime?
   A. 15       B. 17       C. 18       D. 21
   1. _____

2. Find the prime factorization of 18.
   F. $2 \times 9$       G. $3 \times 6$       H. $2^2 \times 3$       J. $2 \times 3^2$
   2. _____

3. Factor $70xy$.
   A. $2 \times 5 \times 7$       C. $10 \times 7 \times x \times y$
   B. $2 \times 5 \times 7 \times x \times y$       D. $10 \times 7 \times x \times x \times y$
   3. _____

4. Find the GCF of 18 and 54.
   F. 6       G. 9       H. 12       J. 18
   4. _____

5. Find the GCF of $12t$ and $18t$.
   A. $6t$       B. $3t$       C. 6       D. 3
   5. _____

6. FIELD TRIPS Seventy-two out of the 81 students are going on the field trip.
   Write $\frac{72}{81}$ in simplest form.
   F. $\frac{3}{4}$       G. $\frac{7}{8}$       H. $\frac{24}{27}$       J. $\frac{8}{9}$
   6. _____

7. Write $\frac{4}{5}$ as a decimal.
   A. 8.0       B. 0.8       C. 0.8       D. 0.08
   7. _____

8. Write $1\frac{8}{9}$ as a decimal.
   F. 0.18       G. 1.8       H. 1.8       J. 18.9
   8. _____

9. SHOPPING Coats are on sale for 23% off. Write 23% as a decimal.
   A. 0.023       B. 0.23       C. 2.3       D. 23
   9. _____

10. SPORTS The basketball team has won 50% of their games so far. Write a ratio equivalent to 50%.
    F. 50 per 100       G. $\frac{5}{100}$       H. 5:1,000       J. 5 per 100
    10. _____

11. Which is 0.25 written as a percent?
    A. $\frac{1}{4}$%       B. 2.5%       C. 25%       D. 250%
    11. _____
12. Write 0.125 as a fraction in simplest form.

F. $\frac{1}{8}$  G. $\frac{1}{6}$  H. $\frac{1}{5}$  J. $\frac{3}{8}$  12. ____

13. Write $\frac{3}{5}$ as a percent.

A. 3.5%  B. 6%  C. 35%  D. 60%  13. ____

14. Write 75% as a fraction in simplest form.

F. $\frac{4}{5}$  G. $\frac{3}{4}$  H. $\frac{1}{4}$  J. $\frac{1}{5}$  14. ____

15. Find the LCM of 15, 25, and 125.

A. 375  B. 275  C. 125  D. 5  15. ____

16. Find the LCD for $\frac{11}{12}$ and $\frac{1}{8}$.

F. 96  G. 88  H. 24  J. 12  16. ____

For Questions 17 and 18, replace each ◯ to make a true sentence.

17. $\frac{13}{28}$ ◯ $\frac{17}{30}$

A. >  B. <  C. =  D. ×  17. ____

18. $\frac{7}{12}$ ◯ $\frac{5}{9}$

F. >  G. <  H. =  J. ÷  18. ____

19. Find the least fraction.

A. $\frac{15}{16}$  B. $\frac{4}{5}$  C. $\frac{9}{10}$  D. $\frac{7}{8}$  19. ____

20. **DOGS** Kelly spent 30 minutes in the afternoon walking her dog. What part of one hour did she spend walking the dog? Write the fraction in simplest form.

F. $\frac{30}{60}$  G. $\frac{30}{24}$  H. $\frac{1}{2}$  J. $\frac{5}{4}$  20. ____

**Bonus** Write 24.1464646… using bar notation.  B: ________________
Chapter 4 Test, Form 2A

Write the letter for the correct answer in the blank at the right of each question.

1. Evaluate \( w^2 - 2w + 6 \) for \( w = 1 \). Is the resulting number prime or composite?
   A. 5; composite  B. 5; prime  C. 4; composite  D. 4; prime

2. Find the prime factorization of 182.
   F. \( 2 \times 91 \)  G. \( 14 \times 13 \)  H. \( 2 \times 7 \times 13 \)  J. \( 13 \times 14 \)

3. Factor \( 50p^2q \).
   A. \( 2 \times 5 \times p \times p \times q \)  B. \( 2 \times 5 \times 5 \times p \times q \times q \)
   C. \( 2 \times 5 \times 5 \times p \times q \)  D. \( 2 \times 5 \times 5 \times p \times p \times q \)

4. Find the GCF of 8, 20, and 36.
   F. 2  G. 4  H. 6  J. 8

5. Find the GCF of \( 16x^2 \) and \( 56x \).
   A. \( 8x \)  B. \( 2x \)  C. 8  D. 4

6. HOUSES Thirty-two out of the 72 new houses are white. Write \( \frac{32}{72} \) in simplest form.
   F. \( \frac{16}{36} \)  G. \( \frac{8}{18} \)  H. \( \frac{4}{9} \)  J. \( \frac{1}{2} \)

7. Write \( \frac{7}{25} \) as a decimal.
   A. 2.8  B. 0.28  C. 0.28  D. 0.028

8. Write \( 4\frac{2}{3} \) as a decimal.
   F. 0.46  G. 4.6  H. \( 4\overline{6} \)  J. \( 46\overline{6} \)

9. Write \( 79\frac{1}{2}\% \) as a decimal.
   A. 79.5  B. 7.95  C. 0.795  D. \( 0.79\frac{1}{2} \)

10. SPORTS The soccer team has lost 30\% of its games so far. Write a ratio equivalent to 30\%.
    F. \( \frac{30}{1000} \)  G. 30 per 100  H. 300:10  J. 3:100

11. Which is 0.07 written as a percent?
    A. 0.07\%  B. 0.7\%  C. 7\%  D. 70\%
12. Write 0.82 as a fraction in simplest form.
   F. \( \frac{41}{50} \)  G. \( \frac{8}{10} \)  H. \( \frac{12}{25} \)  J. \( \frac{2}{5} \)

13. **TRAVEL** Seven out of 16 students on the bus live more than 10 miles away from school. What percent is this?
   A. 0.4375%  B. 4.375%  C. 43.75%  D. 437.5%

14. Write 12% as a fraction in simplest form.
   F. \( \frac{1}{12} \)  G. \( \frac{12}{100} \)  H. \( \frac{3}{25} \)  J. \( \frac{6}{50} \)

15. Find the LCM of 17, 27, and 37.
   A. 629  B. 1,887  C. 5,661  D. 16,983

16. Find the LCD for \( \frac{11}{7} \) and \( \frac{3}{8} \).
   F. 14  G. 16  H. 33  J. 56

For Questions 17 and 18, replace each \( \circ \) to make a true sentence.

17. \( \frac{7}{15} \circ \frac{13}{24} \)
   A. >  B. <  C. =  D. +

18. \( \frac{7}{11} \circ \frac{3}{5} \)
   F. >  G. <  H. =  J. +

19. Find the least fraction.
   A. \( \frac{13}{15} \)  B. \( \frac{7}{8} \)  C. \( \frac{2}{3} \)  D. \( \frac{3}{4} \)

20. **RUNNING** Alberto spent 25 minutes running in the afternoon. What part of one hour did he spend running? Find the fraction in simplest form.
   F. \( \frac{25}{24} \)  G. \( \frac{25}{60} \)  H. \( \frac{5}{60} \)  J. \( \frac{5}{12} \)

**Bonus** What percent of 100 is 50?
   B: ____________
Write the letter for the correct answer in the blank at the right of each question.

1. Evaluate \( s^2 - s + 7 \) for \( s = 2 \). Is the resulting number prime or composite?
   A. 7; prime  
   B. 7; composite  
   C. 9; composite  
   D. 9; prime

2. Find the prime factorization of 42.
   F. \( 2 \times 3 \times 7 \)  
   G. \( 3 \times 14 \)  
   H. \( 3^2 \times 7 \)  
   J. \( 2 \times 5^2 \)

3. Factor \( 81r^2s \).
   A. \( 9 \times 9 \times r \times s \)  
   B. \( 3 \times 3 \times 3 \times r \times r \times s \times s \)  
   C. \( 3 \times 3 \times 3 \times 3 \times r \times s \times s \)  
   D. \( 3 \times 3 \times 3 \times r \times r \times s \times s \)

4. Find the GCF of 36, 60, and 72.
   F. 6  
   G. 12  
   H. 18  
   J. 36

5. Find the GCF of \( 48x \) and \( 56x^2 \).
   A. 4  
   B. 8  
   C. \( 8x \)  
   D. \( 4x^2 \)

6. PETS Twelve out of 27 students in the class own more than one pet. Write \( \frac{12}{27} \) in simplest form.
   F. \( \frac{1}{3} \)  
   G. \( \frac{3}{7} \)  
   H. \( \frac{4}{9} \)  
   J. \( \frac{1}{2} \)

7. Write \( \frac{5}{8} \) as a decimal.
   A. 0.0625  
   B. 0.625  
   C. 0.6\overline{25}  
   D. 6.25

8. Write \( 3\frac{4}{9} \) as a decimal.
   F. 0.34  
   G. 3.4  
   H. 3.\overline{4}  
   J. 34.\overline{4}

9. Write \( 61\frac{1}{2}\% \) as a decimal.
   A. 0.615  
   B. 0.61\frac{1}{2}  
   C. 6.15  
   D. 61.5

10. SPORTS The lacrosse team has won 90% of its games so far. Write a ratio equivalent to 90%.
    F. 10 per 100  
    G. 90 per 100  
    H. 9:100  
    J. \( \frac{90}{1000} \)

11. Which is 0.01 written as a percent?
    A. 99%  
    B. 10%  
    C. 1%  
    D. 0.1%
12. Write 0.75 as a fraction in simplest form.

   F. \( \frac{7}{5} \)  G. \( \frac{75}{100} \)  H. \( \frac{3}{4} \)  J. \( \frac{1}{2} \)  

13. **SALES** The sunglasses Lindsay wants to buy are three-fourths off the regular price. What percent is this?

   A. 0.75  B. 7.5  C. 75%  D. 750%  

14. Write 8% as a fraction in simplest form.

   F. \( \frac{2}{25} \)  G. \( \frac{8}{100} \)  H. \( \frac{4}{50} \)  J. \( \frac{80}{100} \)  

15. Find the LCM of 12, 16, and 24.

   A. 24  B. 48  C. 96  D. 192  

16. Find the LCD for \( \frac{1}{4} \) and \( \frac{2}{3} \).

   F. 4  G. 7  H. 12  J. 18  

For Questions 17 and 18, replace each * to make a true sentence.

17. \( \frac{4}{9} \bullet \frac{7}{12} \)

   A. >  B. <  C. =  D. –  

18. \( \frac{4}{5} \bullet \frac{2}{3} \)

   F. >  G. <  H. =  J. +  

19. Find the least fraction.

   A. \( \frac{5}{7} \)  B. \( \frac{7}{8} \)  C. \( \frac{11}{15} \)  D. \( \frac{7}{10} \)  

20. **LAWN CARE** Erik spent 55 minutes in the morning mowing the lawn.

   What part of one hour did he spend mowing the lawn? Find the fraction in simplest form.

   F. \( \frac{5}{6} \)  G. \( \frac{55}{60} \)  H. \( \frac{11}{12} \)  J. \( \frac{55}{24} \)  

**Bonus** What percent of 100 is 5?
1. Evaluate \( x^2 + x + 3 \) for \( x = 0, 1, \) and 2. Are the resulting numbers prime or composite?

Find the prime factorization of each number.

2. 102

3. 378

Find the GCF of each set of numbers or algebraic expressions.

4. 21, 63

5. 72, 84, 132

6. \( 30a, 42a^2 \)

7. JOBS Corey worked part-time 39 weeks this year. What fraction of the year, in simplest form, did he work?

8. WEATHER It rained 6 out of the last 18 days. What fraction of the last 18 days, in simplest form, did it rain?

Write each fraction, mixed number, or percent as a decimal. Use bar notation if the decimal is a repeating decimal.

9. \( \frac{8}{9} \)

10. \( 6\frac{3}{4} \)

11. 78%

Write each decimal or percent as a fraction in simplest form.

12. 0.02

13. 68%
Write each ratio or decimal as a percent.

14. 36.1 per 100
15. 0.19

16. COMPUTERS Four fifths of the students have a computer in their bedroom. What percent is this?

Find the LCM of each set of numbers.

17. 18, 30, 45
18. 21, 35

Find the LCD for each pair of fractions.

19. \( \frac{9}{15}, \frac{8}{17} \)
20. \( \frac{6}{17}, \frac{19}{51} \)

For Questions 21–23, replace each \( \bullet \) with \(<, >, \text{ or } =\) to make a true sentence.

21. \( \frac{8}{13} \bullet \frac{5}{17} \)
22. \( \frac{10}{15} \bullet \frac{5}{14} \)
23. \( \frac{9}{19} \bullet \frac{19}{39} \)

24. MEASUREMENT Order the lengths \( \frac{1}{4} \) inch, 0.5 inch, and \( \frac{10}{25} \) inch from least to greatest.

25. NEWSPAPER Richard surveyed his class and found that 14 out of 26 students read the sports section of the newspaper first. Write the ratio as a fraction in simplest form and as a decimal to the nearest thousandth.

Bonus Replace \( \bullet \) with a prime factor to make a true sentence: \( 2 \times \bullet \times 7^2 = 294. \)
1. Evaluate $y^2 + 2y + 1$ for $y = 0, 1,$ and $2$. Are the resulting numbers prime or composite?

Find the prime factorization of each number.

2. 72

3. 98

Find the GCF of each set of numbers or algebraic expressions.

4. 48, 56

5. 20, 36, 48

6. $32z, 42z^3$

7. TRANSPORTATION Twenty-eight of the 48 seats on the bus are taken. What fraction is this in simplest form?

8. WEATHER It rained 63 out of the last 102 days. What fraction of the last 102 days, in simplest form, did it rain?

Write each fraction, mixed number, or percent as a decimal. Use bar notation if the decimal is a repeating decimal.

9. $\frac{3}{11}$

10. $7\frac{3}{5}$

11. 84%

Write each decimal or percent as a fraction in simplest form.

12. 0.06

13. 88%
Write each ratio or decimal as a percent.

14. 73.2 per 100
15. 0.27

16. **BIRTHDAYS** Twenty-six out of the fifty students have birthdays during the summer. What percent is this?

Find the LCM of each set of numbers.

17. 24, 30, 360
18. 36, 96

Find the LCD for each pair of fractions.

19. \( \frac{3}{22}, \frac{7}{55} \)
20. \( \frac{9}{10}, \frac{11}{25} \)

For Questions 21–23, replace each \( \bullet \) with <, >, or = to make a true sentence.

21. \( \frac{7}{9} \bullet \frac{15}{17} \)
22. \( \frac{7}{8} \bullet \frac{8}{10} \)
23. \( \frac{7}{11} \bullet \frac{35}{55} \)

24. **MEASUREMENT** Order the lengths \( \frac{7}{8} \text{ feet}, 0.8 \text{ feet}, \) and \( \frac{15}{16} \text{ feet} \) from least to greatest.

25. **MOVING** Louisa surveyed her class and found that 10 out of 24 students had moved to a new city before they started school. Write the ratio as a fraction in simplest form and as a decimal to the nearest thousandth.

**Bonus** Replace \( \square \) with a prime factor to make a true sentence: \( 2^2 \times \square \times 11 = 220. \)
1. Evaluate $6x^2 + 3x + 1$ for $x = 0, 1, 2$. Are the resulting numbers prime or composite?

Find the prime factorization of each number.

2. 51

3. 520

Find the GCF of each set of numbers or algebraic expressions.

4. 49, 84

5. 30, 54, 102

6. $12bc$, $4b^2$, $52b^2c$

7. Write $\frac{14}{63}$ as a fraction in simplest form.

8. **SPORTS** Forty-four of the 121 athletes have already paid their fees. What fraction of athletes, in simplest form, have paid their fees?

Write each fraction, mixed number, or percent as a decimal. Use bar notation if the decimal is a repeating decimal.

9. $\frac{17}{24}$

10. $3 \frac{9}{11}$

11. 5.7%

12. Write 0.94 as a fraction in simplest form.

13. **WEATHER** There is a 46% chance of rain today. Write this as a fraction in simplest form.
Chapter 4 Test, Form 3 (continued)

Write each ratio or decimal as a percent.

14. \(33\frac{1}{3}:100\)

15. 0.3025

16. MONEY Wesley saved $16 of the $25 he earned cutting grass. What percent is this?

Find the LCM of each set of numbers.

17. 21, 49

18. 12, 24, 96

Find the LCD for each pair of fractions.

19. \(\frac{1}{6}, \frac{5}{21}\)

20. \(\frac{4}{9}, \frac{15}{36}\)

For Questions 21–23, replace each \(\bullet\) with <, >, or = to make a true sentence.

21. \(\frac{4}{7} \bullet \frac{5}{8}\)

22. \(\frac{10}{15} \bullet \frac{8}{16}\)

23. \(\frac{3}{5} \bullet \frac{12}{20}\)

24. MEASUREMENT Order the lengths \(\frac{19}{5}\) feet, 4.0625 feet, and \(3\frac{11}{12}\) feet from least to greatest.

25. MUSIC Paris practiced playing the trombone for 1 hour and 24 minutes. Write the time Paris spent practicing in hours as a decimal.

Bonus 29% of 100 is what number?
Chapter 4
Extended-Response Test

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem. If necessary, record your answer on another piece of paper.

1. A community is providing a gardening area for its residents. The area will be laid out in individual garden plots as shown below.

![Diagram of garden area]

a. The garden area will be fenced. The fence posts in front are to be equally spaced along the entire length with posts at the corners of each garden plot. Would a spacing of 3 feet between posts work? Explain.

b. Find the prime factorizations of 30 and 24. Find the common factors of 30 and 24. Explain each step.

c. What is the longest spacing between posts that can be used? What is this distance called? List the multiples of the widths of each size lot. What is the LCM?

d. If the fence posts are to be equally spaced along the width of the garden area with posts at the corners of each garden plot, what spacing would you recommend? Why?

2. Many rates are used in baseball to indicate a player’s or team’s performance.

a. A player’s batting average is the ratio of the number of hits to the number of official times at bat. Julian has 27 hits in 72 times at bat. Write his batting average as a fraction, as a decimal, and as a percent. Show and explain your work.

b. Pitcher Pedro Sanchez won 125 of his games during his career. If he pitched 200 games, explain how you would use mathematics to show his career performance. Show his career performance as a percent, as a decimal, and as a fraction.
Part 1: Multiple Choice

Instructions: Fill in the appropriate circle for the best answer.

1. Solve \( c = 14 + 21 \). (Lesson 1-7)
   A 7  B 34  C 35  D 36
   1. ○ ○ ○ ○

2. Which of these correctly orders the integers from greatest to least? (Lesson 2-2)
   F 7, 3, 0, -8, -1  H -8, -1, 0, 3, 7
   G -8, 7, 3, -1, 0  J 7, 3, 0, -1, -8
   2. ○ ○ ○ ○

3. Find \(-31 + (-5) + 6\). (Lesson 2-4)
   A 42  B -20  C -30  D -32
   3. ○ ○ ○ ○

4. Which of the following equations describe the pattern in the table at the right relating \( x \) and \( y \). (Lesson 2-6)
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   1 & 5 \\
   2 & 7 \\
   3 & 9 \\
   4 & 11 \\
   5 & 13 \\
   \end{array}
   \]
   F \( y = 3x + 2 \)  H \( y = 4x - 1 \)
   G \( y = 2x + 3 \)  J \( y = 3x - 2 \)
   4. ○ ○ ○ ○

5. Sancho burns 200 calories when he works out for 30 minutes. How many calories will he burn if he works out 5 days a week? (Lesson 2-7)
   A 1,000  B 600  C 235  D 150
   5. ○ ○ ○ ○

6. The quotient of a number and 4 is -8. Find the number. (Lesson 2-8)
   F -32  G 22  H 24  J 212
   6. ○ ○ ○ ○

7. What is the value of \( r \) if \( 16r = -62.4 \)? (Lesson 1-7)
   A -78.4  B -39  C -3.9  D 3.9
   7. ○ ○ ○ ○

8. Solve \( 4g + 2 = 10 \). (Lesson 3-5)
   F 2  G 3  H 4  J 8
   8. ○ ○ ○ ○

9. The area of the rectangle is 287.02 square centimeters. What is the length? (Lesson 3-6)
   A 25.4 cm  B 50.8 cm  C 90.2 cm  D 275.72 cm
   9. ○ ○ ○ ○
10. The graph of $y = -2x + 5$ goes through which point? (Lesson 3-7)
   F Q(0, -5)  H S(2, 1)
   G R(1, -3)  J T(3, 1)

11. Which expression represents the prime factorization of 132? (Lesson 4-1)
   A $2 \times 3^2 \times 11$  C $2 \times 6 \times 11$
   B $4 \times 33$  D $2^2 \times 3 \times 11$

12. Write 0.84 as a fraction in simplest form. (Lesson 4-5)
   F $\frac{21}{25}$  H $\frac{84}{100}$
   G $\frac{42}{50}$  J $\frac{42}{5}$

13. Find the LCM of 8, 12, and 28. (Lesson 4-8)
   A 168  C 28
   B 84  D 4

14. The area of the rectangle. (Lesson 3-6)
   F 33.2 mm$^2$  H 57.2 mm$^2$
   G 42.4 mm$^2$  J 220.8 mm$^2$

15. Which is a true sentence? (Lesson 4-9)
   A $\frac{2}{3} < \frac{5}{8}$  C $0.38 = \frac{3}{8}$
   B $4\frac{1}{5} > 4.15$  D $\frac{2}{3} > 70\%$

16. Evaluate $7^4$. (Lesson 1-2)
   F 28  H 1,742
   G 74  J 2,401

17. What is $\frac{36}{30}$ written as a decimal? (Lesson 4-5)
   A 0.6  C 6.5
   B 1.2  D 36.30
18. Evaluate $| -108 |$. (Lesson 1-2)

19. Evaluate the expression $2d + 5f$ if $d = 4$ and $f = 2$. (Lesson 1-6)

20. Graph and label points $A(2, -2)$ and $M(-3, -1)$. (Lesson 2-3)

21. The temperatures in California have varied from $45^\circ F$ to $122^\circ F$. Find the difference in temperatures. (Lesson 2-5)

22. Solve $-6t = 1.8$. (Lesson 3-3)

23. Factor $35a^2b^2$. (Lesson 4-1)

24. Find the greatest common factor of 42, 56, and 91. (Lesson 4-2)

25. Use the numbers $\frac{9}{10}$, 0.99, $\frac{100}{99}$, and 99.99%. (Lesson 4-9)
   a. Order the numbers from least to greatest.
   b. Which ratio is closest to 1? Explain.

Part 2: Short Response

Instructions: Write answers to short response in the space provided.
Read the introduction at the top of page 181 in your textbook.

Write your answers below.

1. Using your grid paper, draw as many different rectangles as possible containing 3, 4, 5, 6, 7, 8, 9, and 10 squares. See Teacher Wraparound Edition, Chapter 5 Answer Appendix for possible drawings.

2. Which number of squares can be drawn in only one rectangle? In more than one rectangle?

3, 5, 7; 4, 6, 8, 9, 10

3. What is the difference between a prime and a composite number?

A prime number is a whole number with only two factors, 1 and itself. A composite number is a whole number greater than one that has more than two factors.

4. How do you know when a factor tree is complete?

A factor tree is complete when you have a row of prime factors.

5. Find the prime factorization of 28 using either method shown in Example 3.

\[ 2 \times 2 \times 7 = 28 \]

6. How can an algebraic expression be factored?

An algebraic expression can be factored as a product of prime numbers and variables.

Remember What You Learned

7. Describe in your own words how to use a factor tree to find the prime factorization of a number. Include an example as an explanation.

Answers may vary.

NAME ___________________________ DATE ______________ PERIOD _____

Lesson Reading Guide

Chapter 4

Anticipation Guide

Before you begin Chapter 4

• Read each statement.
• Decide whether you Agree (A) or Disagree (D) with the statement.
• Write A or D in the first column OR if you are not sure whether you agree or disagree, write NS (Not Sure).

After you complete Chapter 4

• Reread each statement and complete the last column by entering an A (Agree) or a D (Disagree).
• Did any of your opinions about the statements change from the first column?
• For those statements that you mark with a D, use a separate sheet of paper to explain why you disagree. Use examples, if possible.

1. A composite number is a number with two or more digits.
   D

2. A factor tree is used to find the prime factorization of a number.
   A

3. The greatest common factor of two prime numbers is 1.
   A

4. Two fractions are equivalent only if they have the same numerator and the same denominator.
   D

5. A fraction is in simplest form only when the greatest common factor of the numerator and denominator is 1.
   A

6. \( \frac{1}{2} \) is equivalent to 1.2.
   D

7. To write a fraction as a decimal, divide the denominator into the numerator.
   A

8. The following are all ratios: 12 out of 65, 12:65, and \( \frac{12}{65} \).
   A

9. To write \( \frac{23}{55} \) as a percent, first you must write the fraction in simplest form.
   D

10. Moving a decimal point two places to the right is the same as dividing by 100.
    D

11. To find the least common multiple of two numbers, make a list of multiples of both numbers.
    A

12. When comparing two fractions, first rewrite the fractions with a common denominator.
    A
**Example 1**

Determine whether each number is prime or composite.

a. 11  b. 24

a. The number 11 has only two factors, 1 and 11, so it is prime.

b. The number 24 has 8 factors, 1, 2, 3, 4, 6, 8, 12, and 24. So, it is composite.

**Example 2**

Determine the prime factorization of 48.

Use a factor tree.

```
    48
   /  \  \
  2   24
   \  /  \ 
   3  8
      /  \  \
     3  2
```

The prime factorization of 48 is $2^4 \cdot 3$.

**Exercises**

Determine whether each number is prime or composite.

1. 27 composite  
2. 31 prime  
3. 46 composite  
4. 53 prime  
5. 11 prime  
6. 72 composite  
7. 17 prime  
8. 51 composite

Determine the prime factorization of the following numbers.

9. 64 $2^6$  
10. 100 $2^2 \cdot 5^2$  
11. 45 $3^2 \cdot 5$  
12. 81 $3^4$  
13. 425 $5^2 \cdot 17$  
14. 82 $2 \cdot 41$  
15. 90 $2 \cdot 3^2 \cdot 5$  
16. 63 $3^2 \cdot 7$  
17. 236 $2^2 \cdot 59$  
18. 12 $2^2 \cdot 3$  
19. 110 $2 \cdot 5 \cdot 11$  
20. 46 $2 \cdot 23$  
21. 84 $2^2 \cdot 3 \cdot 7$
Answers (Lesson 4-1)

Prime Factorization

1. **AGE**
   The average life expectancy in the United States is now 77.6 years. Round to the nearest whole number, and write it as a product of primes.
   - 78 = 2 \cdot 3 \cdot 13

2. **FOOTBALL**
   A football team’s record for the season is 12 wins and 4 losses. Write their record as a product of primes.
   - 12 = 2^2 \cdot 3
   - 4 = 2^2

3. **BASKETBALL**
   The average height of players in the NBA is 6 feet 7 inches. Write this height in inches as a product of primes.
   - 79 = 79

4. **TRAVELING**
   The distance between Washington, D.C. and Chicago, IL is about 590 miles by air. Write this distance as a product of primes.
   - 590 = 2 \cdot 5 \cdot 59

5. **SCIENCE**
   There are 118 elements in the Periodic Table. List all of the factors of 118. What type of number is this?
   - 1, 2, 59, 118
   - Composite

6. **READING**
   A copy of *A Tale of Two Cities*, the classic written by Charles Dickens, has about 530 pages. Write this as a product of primes.
   - 530 = 2 \cdot 5 \cdot 53

**Determining whether each number is prime or composite.**

1. 45: Composite
2. 17: Prime
3. 21: Composite
4. 51: Composite
5. 6: Prime
6. 71: Prime
7. 87: Composite
8. 37: Prime
9. 47: Prime
10. 51: Composite
11. 11: Prime
12. 71: Prime
13. 3: Prime
14. 27: Composite
15. 47: Prime

**Finding the prime factorization of each number.**

10. 88 = 2^3 \cdot 11
11. 39 = 3 \cdot 13
12. 75 = 3 \cdot 5^2
13. 124 = 2^2 \cdot 31
14. 105 = 3 \cdot 5 \cdot 11
15. 225 = 3^2 \cdot 5^2
16. 180 = 2^2 \cdot 3 \cdot 5
17. 51 = 3 \cdot 17
18. 91 = 7 \cdot 13
19. 180 = 2^2 \cdot 3^2 \cdot 5
20. 150 = 2 \cdot 3 \cdot 5^2
21. 11pc
22. 21 = 3 \cdot 7
23. 34 = 2 \cdot 17
24. 24 = 2^3 \cdot 3
25. 27 = 3^3
26. 30 = 2 \cdot 3 \cdot 5
27. 36 = 2^2 \cdot 3^2
28. 44 = 2^2 \cdot 11
29. 72 = 2^3 \cdot 3^2
30. 125 = 5^3
31. 144 = 2^4 \cdot 3^2
32. 225 = 3^2 \cdot 5^2
33. 400 = 2^4 \cdot 5^2
34. 500 = 2^2 \cdot 5^3
35. 750 = 2 \cdot 3 \cdot 5^3
36. 1,000 = 2^3 \cdot 5^3
37. 1,800 = 2^3 \cdot 3 \cdot 5^3
38. 3,600 = 2^4 \cdot 3^2 \cdot 5^2
39. 18,000 = 2^4 \cdot 3^2 \cdot 5^3
40. 27,000 = 2^1 \cdot 3^3 \cdot 5^3

**Replacing each with prime factors to make a true sentence.**

25. 180 = 2^2 \cdot 3 \cdot 5
26. 360 = 2^3 \cdot 3^2
27. 720 = 2^4 \cdot 3 \cdot 5
28. 1,080 = 2^3 \cdot 3^3 \cdot 5
29. 2,160 = 2^4 \cdot 3^3
30. 3,780 = 2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 11
31. 7,290 = 2 \cdot 3^4 \cdot 5 \cdot 17
32. 18,090 = 2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 17
33. 36,288 = 2^{10} \cdot 3 \cdot 5 \cdot 11
34. 72,576 = 2^{10} \cdot 3^2 \cdot 11
35. 145,800 = 2^3 \cdot 3^4 \cdot 5^2 \cdot 11
36. 281,760 = 2^{12} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11
37. 563,520 = 2^{12} \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13
38. 1,127,040 = 2^{12} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13
39. 2,254,560 = 2^{12} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13
40. 4,509,120 = 2^{12} \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13
Chapter 4

NAME ________________________________________ DATE ______________ PERIOD _____

**Enrichment**

**Perfect Numbers**
A positive integer is **perfect** if it equals the sum of its factors that are less than the integer itself.
If the sum of the factors (excluding the integer itself) is greater than the integer, the integer is called **abundant**.
If the sum of the factors (excluding the integer itself) is less than the integer, the integer is called **deficient**.

Example 1
Use a spreadsheet to find the greatest common factor of 32 and 52.

Step 1
The factors of 28 (excluding 28 itself) are 1, 2, 4, 7, and 14.

Step 2
In cell A3, enter an equals sign followed by MOD(A1,A2). Then press ENTER.

Step 3
Click on the bottom right corner of cell A3 and drag it down several rows.

So, the greatest common factor of 32 and 52 is 4.

Complete the table to classify each number as perfect, abundant, or deficient.

<table>
<thead>
<tr>
<th>Number</th>
<th>Divisors (Excluding the Number Itself)</th>
<th>Sum</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>14, 1, 2, 7</td>
<td>10</td>
<td>deficient</td>
</tr>
<tr>
<td>2.</td>
<td>6, 1, 2, 3</td>
<td>6</td>
<td>perfect</td>
</tr>
<tr>
<td>3.</td>
<td>12, 1, 2, 3, 4, 6</td>
<td>16</td>
<td>abundant</td>
</tr>
<tr>
<td>4.</td>
<td>20, 1, 2, 4, 5, 10</td>
<td>22</td>
<td>abundant</td>
</tr>
<tr>
<td>5.</td>
<td>10, 1, 2, 5</td>
<td>8</td>
<td>deficient</td>
</tr>
</tbody>
</table>

Show that each number is perfect.

6. 496
1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496

7. 8,128
1 + 2 + 4 + 8 + 16 + 32 + 64 + 127 + 254 + 508 +
1,016 + 2,032 + 4,064 = 8,128

8. CHALLENGE
33,550,336
1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 +
256 + 512 + 1,024 + 2,048 + 4,096 + 8,191 + 16,382 +
32,766 + 65,528 + 131,056 + 262,112 + 524,224 +
1,048,448 + 2,096,896 + 4,193,792 + 8,387,584 +
16,775,168 = 33,550,336

Find the greatest common factor of each pair of numbers.

1. 15 and 50
2. 27 and 32
3. 100 and 29

4. 211 and 15
5. 201 and 18
6. 18 and 81

7. 55 and 200
8. 33 and 121
9. 16 and 120

10. 14 and 21
11. 13 and 52
12. 90 and 1800

13. 225 and 15
14. 169 and 13
15. 80 and 64

Answers (Lesson 4-1)
Answers (Lesson 4-2)

Example 1

Find the GCF of 72 and 108 by listing factors.

Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
Factors of 108: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108

Common factors: 1, 2, 3, 4, 6, 9, 12, 18, 36

The GCF of 72 and 108 is 36.

Example 2

Find the GCF of 42 and 60 using prime factors.

Method 1
Write the prime factorization.

Method 2
Divide by prime numbers.

Divide both 42 and 60 by 2.

36
2
18
3
6
2
3
2

The common prime factors are 2 and 3. The GCF of 42 and 60 is 2 × 3, or 6.

Exercises

Find the GCF of each set of numbers.

1. 18, 30
2. 60, 45
3. 24, 72
4. 32, 48
5. 100, 30
6. 54, 36
7. 3, 9, 5
8. 4, 20, 24

The greatest common factor (GCF) of two or more numbers is the largest number that is a factor of each number. The GCF of prime numbers is 1.
Find the GCF of each set of numbers.

1. 14, 20
2. 16, 42
3. 8, 18
4. 24, 36
5. 72, 22
6. 77, 15
7. 32, 80
8. 90, 120
9. 45, 30
10. 12, 62
11. 15, 27
12. 21, 28
13. 12, 20, 26
14. 15, 20, 25
15. 60, 72, 36
16. 32, 48, 64
17. 36, 8, 30
18. 28, 56, 42
19. 80, 110, 90
20. 9, 25, 49

Find the GCF of each set of algebraic expressions.

21. 2ab, 14b
22. 20a^2, 36a
23. 15ab, 5b
24. 3ax^2, 8ay
25. Find the GCF of 2^3 \times 3^2 \times 5 and 2^2 \times 3 \times 5^2. 2^2 \times 3 \times 5 or 60

Find two numbers whose GCF is the given number.

21. 10
22. 8
23. 14

Sample answer: 20, 50 Sample answer: 24, 32 Sample answer: 56, 70

24. SPORTS CARDS Jason wants to organize his sports cards in packets for each type of sport. Each packet has the same number of cards. If he has 24 baseball cards, 60 hockey cards, and 48 football cards, find the greatest number of cards in each packet. 12 cards

25. FORESTRY A forest ranger needs to remove three tree trunks by cutting the trunks into equal lengths. If the lengths of the tree trunks are 6 feet, 8 feet, and 12 feet, what is the length of the longest log that can be cut? 2 feet
This arrangement of numbers is called Sundaram's Sieve. Like the Sieve of Eratosthenes, Sundaram's arrangement can be used to find prime numbers.

Here's how to use Sundaram's Sieve to find prime numbers. If a number, \( n \), is not in the sieve, then \( \frac{2n}{1} + 1 \) is a prime number. If a number, \( n \), is in the sieve, then \( \frac{2n}{1} + 1 \) is not a prime number.

32 is in the sieve.

\[ \frac{2 \times 32}{1} + 1 = 65 \]

65 is not prime.

35 is not in the sieve.

\[ \frac{2 \times 35}{1} + 1 = 71 \]

71 is prime.

1. Does the sieve give all primes up to 99? all the composites?
   All primes except 2; only 22 of the composites.

2. Sundaram's Sieve is constructed from arithmetic sequences. Describe the pattern used to make the first row.
   Start with 4 and add 3 each time.

3. How is the first column constructed?
   It is the same as the first row.

4. How are the second through fifth rows constructed?
   Arithmetic sequences using 5, 7, 9, and 11.

5. How would you add a sixth row to the sieve?
   Start with 19 and add 13 each time.

6. Use Sundaram's Sieve to find 5 four-digit prime numbers. You will need to add more numbers to the sieve to do this.
   Answers will vary.
Chapter 4

Study Guide and Intervention

Problem-Solving Investigation: Make an Organized List

When solving problems often times it is useful to make an organized list. By doing so you can see all the possible solutions to the problem being posed.

**Example 1**

LUNCH

Walnut Hills School has a deli line where students are able to select a meat sandwich, a side, and fruit. Meat choices are ham or turkey. The side choices are pretzels or chips. Fruit options are an apple or a pear. How many different combinations are possible?

**Explore**

You know that students can choose a sandwich, a side, and fruit. There are 2 meat choices, 2 side choices, and 2 fruit choices. You need to find all possible combinations.

**Plan**

Make an organized list.

**Solve**

<table>
<thead>
<tr>
<th>Meat</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side</td>
<td>Pretzel</td>
<td>Pretzel</td>
<td>Chips</td>
<td>Chips</td>
<td>Pretzel</td>
<td>Pretzel</td>
<td>Chips</td>
<td>Chips</td>
</tr>
<tr>
<td>Fruit</td>
<td>Apple</td>
<td>Apple</td>
<td>Pear</td>
<td>Pear</td>
<td>Apple</td>
<td>Apple</td>
<td>Pear</td>
<td>Pear</td>
</tr>
</tbody>
</table>

There are 8 possibilities.

**Check**

Draw a tree diagram to check the result.

1. **BAKING** Virginia and Robert have 1 dozen of each of the following types of cookies: chocolate chip, oatmeal raisin, snickerdoodles, and shortbread. If they want to divide the cookies into packages of two dozen, with one dozen of each of two types of cookies per package, how many different ways can they group the cookies?
   6

2. **NUMBER THEORY** How many different two-digit numbers can be made using the digits 2, 9, 6, and 3?
   12

3. **FOOD** Takanae is ordering lunch at a deli and is trying to decide what she would like on her sandwich. She has her choice of turkey, ham, or roast beef and a choice of cheddar, swiss, or muenster cheese. How many combinations of sandwich could she choose assuming that each sandwich has one type of meat and one type of cheese?
   9

4. **TELEPHONES** How many phone numbers are possible for one area code if the first four numbers are 202-1, in that order, and the last three numbers are 1-7-8 in any order?
   6

5. **CLOTHES** Sheila has four different shirts and two skirts with her on a business trip. How many different outfits can she create?
   8

6. **SPORTS** Juan and Andrew are planning the schedule for a softball tournament. If there are 6 teams, how many different pairings could they make for the first tournament game?
   15

**Exercises**

1. Susan has 3 shirts; red, blue, and green; 2 pants; jeans and khakis; and 3 shoes; white, black, and tan, to choose from for her school outfit. How many different outfits can she create?
   **18 different outfits**

2. The Motor Speedway is awarding money to the first two finishers in their annual race. If there are four cars in the race numbered 1 through 4, how many different ways can they come in first and second?
   **12 different ways**

Chapter 4

23

Glencoe California Mathematics, Grade 6
Problem-Solving Investigation: Make An Organized List

For Exercises 1 and 2, solve each problem by making an organized list.

1. VACATION Keesha, Kacy, and their parents sit in different seats in the car while driving to their grandparents for vacation. If only the parents take turns driving, how many different ways can all four people sit in the car with 2 front and 2 back seats?
   - 12 different ways

2. PIZZA Everyone at the table likes pepperoni, sausage, onions, and black olives on pizza. List the different possibilities of ordering a 2-topping pizza.
   - pepperoni and sausage
   - pepperoni and onion
   - pepperoni and black olives
   - sausage and onion
   - sausage and black olives
   - onion and black olives

Use any strategy to solve Exercises 3 and 4. Some strategies are shown below.

PROBLEM-SOLVING STRATEGIES
- Use the four-step plan
- Guess and check
- Work backward
- Make an organized list

3. NUMBER SENSE A number is increased by 12. When this sum is divided by 3, the result is the original number. What is the number?
   - The number is 6.

4. COINS Three coins are tossed: a quarter, a nickel, and a dime. Complete the table showing the 8 different ways the coins could land by using H for heads and T for tails.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Nickel</th>
<th>Dime</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
<td>T</td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td>H</td>
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<td>H</td>
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<td>T</td>
<td>H</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>H</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

Select the Operation

For Exercises 5 and 6, select the appropriate operation(s) to solve the problem. Justify your solution(s) and solve the problem.

5. MEASUREMENT Eight furlongs is equal to one mile. If a mile is 5,280 feet, how many feet are in 5 furlongs?
   - divide and multiply
   - 5,280 - 8 = 660
   - 660 - 5 = 3,300
   - 5 furlongs equals 3,300 feet.

6. TIME Juanita is trying to get used to waking up earlier in the morning. She wakes up at 8:30 now, but wants to wake up at 7:15. If she wakes up five minutes earlier each morning, how many mornings will it be until she wakes up at 7:15?
   - add and subtract
   - 14 + 12 + 15 = 41
   - 53 - 41 = 12
   - 12 hours

7. CLOTHES Carly is taking a vacation with her family and packs three pairs of shorts and four tops. How many different outfits can she make if she wears each top with each pair of shorts?
   - 12 outfits

8. FOOD Carlos is making a fruit salad and wants to use only 2 types of fruit. If he has blueberries, strawberries, grapes, oranges, and bananas on hand, how many combinations could he make?
   - 10 combinations
Simplifying Fractions

Lesson 4–4

Write \(\frac{35}{64}\) in simplest form.

First, find the GCF of the numerator and denominator.

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

The GCF of 36 and 54 is 18.

Then, divide the numerator and the denominator by the GCF.

\[
\frac{35}{64} \rightarrow \frac{35 \div 18}{64 \div 18} = \frac{23}{34}
\]

So, \(\frac{35}{64}\) written in simplest form is \(\frac{23}{34}\).

Write \(\frac{1}{8}\) in simplest form.

Find the GCF of the numerator and the denominator.

Factors of 8: 1, 2, 4, 8
Factors of 12: 1, 2, 3, 4, 6, 12

The GCF of 8 and 12 is 4.

\[
\frac{1}{8} \rightarrow \frac{1 \div 4}{8 \div 4} = \frac{1}{2}
\]

So, \(\frac{1}{8}\) written in simplest form is \(\frac{1}{2}\).

Write each fraction in simplest form.

1. \(\frac{47}{22}\)
2. \(\frac{1}{7}\)
3. \(\frac{46}{58}\)
4. \(\frac{1}{5}\)
5. \(\frac{12}{19}\)
6. \(\frac{18}{75}\)

Fractions that have the same value are called equivalent fractions. A fraction is in simplest form when the GCF of the numerator and denominator is 1.

Example 1

Example 2

Exercises

6NS2.4

Get Ready for the Lesson

Complete the Mini Lab at the top of page 192 in your textbook. Write your answers below.

1. Write a fraction to describe each figure: \(\frac{1}{4}\); \(\frac{5}{2}\)

2. Based on the figures, what can you conclude about the fractions?

They are equal.

Read the Lesson

1. Write a fraction to describe each figure: \(\frac{4}{10}\)

2. Based on the figures, what can you conclude about the fractions?

They are equal.

3. How do you find the simplest form of a fraction?

Sample answer:

1) Find the GCF of the numerator and denominator. 2) Divide the numerator and denominator by the GCF.

4. When you find the simplest form of a fraction, how can you check to make sure your answer is correct?

Multiply the numerator and denominator of the answer by the GCF of the original numerator and denominator. If you get the original fraction, your answer is correct.

5. Use canceling to simplify the fraction \(\frac{2}{3}\).\(\frac{1}{17}\)

\[
\frac{2}{3} \times \frac{1}{17} = \frac{2 \times 1}{3 \times 17} = \frac{2}{51}
\]

Remember, What You Learned

6. Use a collection of rectangles like the one in the Mini Lab to show how to write \(\frac{23}{34}\) in simplest form.
Skills Practice

Write each fraction in simplest form.

1. \( \frac{12}{30} \)
2. \( \frac{20}{35} \)
3. \( \frac{49}{78} \)
4. \( \frac{24}{39} \)
5. \( \frac{14}{21} \)
6. \( \frac{28}{42} \)

Write two fractions that are equivalent to each fraction.


16. \( \frac{3}{6} = \frac{1}{2} \)
17. \( \frac{4}{8} = \frac{1}{2} \)
18. \( \frac{2}{3} = \frac{6}{9} \)
19. \( \frac{14}{21} = \frac{2}{3} \)
20. \( \frac{20}{30} = \frac{2}{3} \)
21. \( \frac{42}{63} = \frac{2}{3} \)
22. \( \frac{48}{72} = \frac{2}{3} \)
23. \( \frac{35}{51} = \frac{5}{9} \)

Practice

Simplifying Fractions

Write each fraction in simplest form.

1. \( \frac{4}{10} = \frac{2}{5} \)
2. \( \frac{6}{20} = \frac{3}{10} \)
3. \( \frac{5}{16} = \frac{5}{16} \)
4. \( \frac{14}{28} = \frac{1}{2} \)

Write two fractions that are equivalent to each fraction.


16. \( \frac{3}{6} = \frac{1}{2} \)
17. \( \frac{4}{8} = \frac{1}{2} \)
18. \( \frac{2}{3} = \frac{6}{9} \)
19. \( \frac{14}{21} = \frac{2}{3} \)
20. \( \frac{20}{30} = \frac{2}{3} \)
21. \( \frac{42}{63} = \frac{2}{3} \)
22. \( \frac{48}{72} = \frac{2}{3} \)
23. \( \frac{35}{51} = \frac{5}{9} \)

22. Measurement

Fifteen inches is what fraction, in simplest form, of a yard? (Lesson 4-4)

23. Money

Thirty-five cents is what fraction, in simplest form, of a dollar?
It can be difficult to understand comparisons of different continents and their populations because the numbers are so large. You can make these comparisons easier to understand by writing them as fractions and using rounding to find an estimated ratio.

For example, the ratio of Asia’s population to North America’s population is \( \frac{3,879,000,000}{501,500,000} \), or \( \frac{3,879}{501.5} \). If you divide both the numerator and denominator by 1,000, you get \( \frac{3.879}{0.502} \), which can be approximated as \( \frac{4}{0.5} \) or about \( \frac{18}{1} \).

1. Approximately what fraction of the world’s land area is found in South America?  
   \( \frac{11}{43} \)

2. Approximately what fraction of the world’s population is found in South America?  
   \( \frac{11}{43} \)

3. Of the first 22 presidents, 8 were from New York. Write the number of presidents from New York as a fraction of the first 22 presidents in simplest form.  
   \( \frac{4}{11} \)

4. Ten hours is what part of a day?  
   \( \frac{5}{12} \)

5. Eighteen inches is what part of a yard?  
   \( \frac{1}{2} \)
Answers (Lesson 4-5)

**Example 1**

Write 0.32 as a fraction in simplest form.

- **Method 1** Use pencil and paper.
  - 0.32 can be written as $\frac{32}{100}$.
  - Simplify by dividing both numerator and denominator by 4 to get $\frac{8}{25}$.

- **Method 2** Use a calculator.
  - The calculator shows $0.32 = \frac{8}{25}$.

**Write each fraction as a decimal. Use bar notation if the decimal is a repeating decimal.**

1. $\frac{1}{8} = 0.125$
2. $\frac{3}{5} = 0.6$
3. $\frac{1}{7} = 0.142857...$
4. $\frac{7}{8} = 0.875$
5. $\frac{11}{35} = 0.3142857...$
6. $\frac{3}{49} = 0.0612244898...$

**Write each decimal as a fraction in simplest form.**

7. $0.14 = \frac{7}{50}$
8. $0.3 = \frac{3}{10}$
9. $0.94 = \frac{47}{50}$

**Get Ready for the Lesson**

1. What fraction of the buildings are between 600 and 900 feet tall?
   - $\frac{9}{10}$

2. Express this fraction using words and then as a decimal.
   - Nine tenths; 0.9

3. What fraction of the buildings are between 710 and 730 feet tall? Express this fraction using words and then as a decimal.
   - Three tenths; 0.3

**Read the Lesson**

4. What is meant by the term place value?
   - Sample answer: The value of the place of a digit in a numeral

5. In place value, what serves as the divider between ones and tenths?
   - The decimal point

6. What is the difference between a terminating decimal and a repeating decimal? Give an example of each.
   - A terminating decimal has an ending, and a repeating decimal does not; 6.25 is a terminating decimal, and 0.7777... is a repeating decimal.

**Remember What You Learned**

7. Work with a partner. Use a local newspaper, a favorite magazine, or the Internet. Find real-world situations that use fractions or decimals. Convert the fractions to decimals and the decimals to fractions. Exchange papers with your partner and correct each other's work.
### Fractions and Decimals

Write each repeating decimal using bar notation.

1. \(0.7353535\ldots\)  
2. \(0.424242\ldots\)  
3. \(2.126126126\ldots\)

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

4. \(0.73\)  
5. \(0.7\)

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

6. \(0.2\)  
7. \(0.16\)

Write each decimal as a fraction or mixed number in simplest form.

8. \(0.4\)  
9. \(0.16\)

Write each decimal as a fraction in simplest form.

10. \(0.8\)  
11. \(0.08\)

One kilometer is approximately 0.62 mile. What fraction represents this length?

12. \(0.62\)  
13. \(0.31875\)

Jake completed a marathon race in 3 hours and 12 minutes. Write Jake’s running time as a decimal.

14. \(3.2\)  
15. \(3.125\)

A marathon is approximately 26.2 miles. What fraction represents this distance?

16. \(\frac{26}{20}\)  
17. \(\frac{26}{20}\)

### Skills Practice

**Fractions and Decimals**

Write each repeating decimal using bar notation.

1. \(\frac{1}{3}\)  
2. \(\frac{1}{8}\)  
3. \(\frac{1}{8}\)

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

4. \(\frac{1}{8}\)  
5. \(\frac{1}{8}\)

Write each decimal as a fraction or mixed number in simplest form.

6. \(\frac{1}{8}\)  
7. \(\frac{1}{8}\)

Write each decimal as a fraction in simplest form.

8. \(\frac{1}{8}\)  
9. \(\frac{1}{8}\)

### Answers (Lesson 4-5)

- **Skills Practice**
  - Fractions and Decimals
    - Write each repeating decimal using bar notation.
    - Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.
    - Write each decimal as a fraction or mixed number in simplest form.
    - Write each decimal as a fraction in simplest form.

- **Chapter 4**
  - **4-5**
    - Fractions and Decimals
      - Write each repeating decimal using bar notation.
      - Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.
      - Write each decimal as a fraction or mixed number in simplest form.
      - Write each decimal as a fraction in simplest form.

- **Lesson 4-5**

- **Answers**
  - Write each repeating decimal using bar notation.
  - Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.
  - Write each decimal as a fraction or mixed number in simplest form.
  - Write each decimal as a fraction in simplest form.

- **Skills Practice**
  - Write each repeating decimal using bar notation.
  - Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.
  - Write each decimal as a fraction or mixed number in simplest form.
  - Write each decimal as a fraction in simplest form.
Answers (Lesson 4-5)

**Word Problem Practice**

1. **BOYS AND GIRLS**
   - There were 6 girls and 2 boys in Mrs. Johnson's math class. Write the number of girls as a fraction of the number of boys. Then write the fraction as a decimal.
   - **Solution:**
     - Number of girls = 6
     - Number of boys = 2
     - Fraction of girls = \( \frac{6}{2} = 3 \)
     - Fraction as a decimal = 3

2. **CATS**
   - In a neighborhood of 72 families, 18 families own one or more cats. Write the number of families who own one or more cats as a fraction. Then write the fraction as a decimal.
   - **Solution:**
     - Number of families = 72
     - Number of families who own cats = 18
     - Fraction of families who own cats = \( \frac{18}{72} = \frac{1}{4} \)
     - Fraction as a decimal = 0.25

**Fractions and Decimals**

- **Writing Repeating Decimals as Fractions**
  - All fractions can be written as decimals that either terminate or repeat. You have learned how to use a power of 10 to write a terminating decimal as a fraction. Below, you will study a strategy to write a repeating decimal as a fraction.

1. **Enrichment**
   - For Exercises 1–4, write each fraction as a decimal. Use bar notation if the decimal is a repeating decimal.
   - **Exercises:**
     - 1. \( \frac{1}{3} \)
     - 2. \( \frac{1}{8} \)
     - 3. \( \frac{1}{9} \)
     - 4. \( \frac{5}{9} \)
   - **Answers:**
     - 1. 0.3
     - 2. 0.125
     - 3. 0.111111111...
     - 4. 0.555555555...

2. **FINAL PDF PROOF**
   - For Exercises 5 and 11, make a prediction about the decimal equivalent of each fraction before writing the fraction as a decimal. Check your work using long division.
   - **Exercises:**
     - 5. \( \frac{1}{7} \)
     - 6. \( \frac{1}{11} \)
     - 7. \( \frac{1}{13} \)
     - 8. \( \frac{1}{19} \)
     - 9. \( \frac{1}{23} \)
     - 10. \( \frac{1}{29} \)
   - **Answers:**
     - 5. 0.142857142857...
     - 6. 0.090909090909...
     - 7. 0.076923076923...
     - 8. 0.0526315789473684...
     - 9. 0.04347826086956521739130434782608695652173913...
     - 10. 0.03448275862068965517241379310344827586206896551724137931...

3. **LINES AND TELEPHONES**
   - Tora took a short trip of 200 miles. He stopped to have lunch 120 miles from the starting point. Write the fraction of the trip he had completed by lunch in simplest form.
   - **Solution:**
     - Distance to lunch = 120 miles
     - Total distance = 200 miles
     - Fraction of trip completed = \( \frac{120}{200} = \frac{3}{5} \)

4. **VOTING**
   - In a recent school election, 20% of the 925 freshmen voted in their class election. Write the fraction of freshmen who voted as a fraction in simplest form.
   - **Solution:**
     - Number of freshmen = 925
     - Number of freshmen who voted = 0.2 * 925 = 185
     - Fraction of freshmen who voted = \( \frac{185}{925} = \frac{37}{185} \)

5. **TRAVEL**
   - Travel checked out a short trip of 600 miles. He stopped to have lunch 240 miles from the starting point. Write the fraction of the trip he had completed by lunch in simplest form.
   - **Solution:**
     - Distance to lunch = 240 miles
     - Total distance = 600 miles
     - Fraction of trip completed = \( \frac{240}{600} = \frac{2}{5} \)

6. **AVERAGE**
   - Ms. Rockwell surveyed her class and found that 12 out of the 30 students chose peaches as their favorite fruit. Write the number of students who chose peaches as a fraction.
   - **Solution:**
     - Number of students = 30
     - Number of students who chose peaches = 12
     - Fraction of students who chose peaches = \( \frac{12}{30} = \frac{2}{5} \)

**Writing Repeating Decimals as Fractions**

- **Writing Repeating Decimals as Fractions**
  - All fractions can be written as decimals that either terminate or repeat. You have learned how to use a power of 10 to write a terminating decimal as a fraction. Below, you will study a strategy to write a repeating decimal as a fraction.

1. **Enrichment**
   - For Exercises 1–4, write each fraction as a decimal. Use bar notation if the decimal is a repeating decimal.
   - **Exercises:**
     - 1. \( \frac{1}{3} \)
     - 2. \( \frac{1}{8} \)
     - 3. \( \frac{1}{9} \)
     - 4. \( \frac{5}{9} \)
   - **Answers:**
     - 1. 0.3
     - 2. 0.125
     - 3. 0.111111111...
     - 4. 0.555555555...

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   - For Exercises 5 and 11, make a prediction about the decimal equivalent of each fraction before writing the fraction as a decimal. Check your work using long division.
   - **Exercises:**
     - 5. \( \frac{1}{7} \)
     - 6. \( \frac{1}{11} \)
     - 7. \( \frac{1}{13} \)
     - 8. \( \frac{1}{19} \)
     - 9. \( \frac{1}{23} \)
     - 10. \( \frac{1}{29} \)
   - **Answers:**
     - 5. 0.142857142857...
     - 6. 0.090909090909...
     - 7. 0.076923076923...
     - 8. 0.0526315789473684...
     - 9. 0.04347826086956521739130434782608695652173913...
     - 10. 0.03448275862068965517241379310344827586206896551724137931...

3. **LINES AND TELEPHONES**
   - Tora took a short trip of 200 miles. He stopped to have lunch 120 miles from the starting point. Write the fraction of the trip he had completed by lunch in simplest form.
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     - Total distance = 200 miles
     - Fraction of trip completed = \( \frac{120}{200} = \frac{3}{5} \)

4. **VOTING**
   - In a recent school election, 20% of the 925 freshmen voted in their class election. Write the fraction of freshmen who voted. Then write the fraction as a decimal.
   - **Solution:**
     - Number of freshmen = 925
     - Number of freshmen who voted = 0.2 * 925 = 185
     - Fraction of freshmen who voted = \( \frac{185}{925} = \frac{37}{185} \)

5. **TRAVEL**
   - Travel checked out a short trip of 600 miles. He stopped to have lunch 240 miles from the starting point. Write the fraction of the trip he had completed by lunch in simplest form.
   - **Solution:**
     - Distance to lunch = 240 miles
     - Total distance = 600 miles
     - Fraction of trip completed = \( \frac{240}{600} = \frac{2}{5} \)

6. **AVERAGE**
   - Ms. Rockwell surveyed her class and found that 12 out of the 30 students chose peaches as their favorite fruit. Write the number of students who chose peaches as a fraction.
   - **Solution:**
     - Number of students = 30
     - Number of students who chose peaches = 12
     - Fraction of students who chose peaches = \( \frac{12}{30} = \frac{2}{5} \)
Get Ready for the Lesson

Read the introduction at the top of page 202 in your textbook.

Write your answers below.

1. For each method, shade a \(10 \times 10\) grid that represents the number of students that chose the method.

2. What fraction of the students chose the Internet?

3. There is more than one way to write a ratio. Write the ratio that compares 4 to 25 in three different ways.
   Sample answer: 4 out of 25, 4:25, and \(4 \div 25\).

4. Write the ratio in Exercise 3 as a percent.
   16%

5. How does having ratios written as percents make it easier to compare amounts? Sample answer: Percents have the same denominator (100), so all you need to do is compare the numerators.

Read the Lesson

2. What fraction of the students chose the Internet?
   \(\frac{17}{50}\)

3. There is more than one way to write a ratio. Write the ratio that compares 4 to 25 in three different ways.
   Sample answer: 4 out of 25, 4:25, and \(\frac{4}{25}\).

4. Write the ratio in Exercise 3 as a percent.
   16%

5. How does having ratios written as percents make it easier to compare amounts? Sample answer: Percents have the same denominator (100), so all you need to do is compare the numerators.

Remember What You Learned

6. Work with a partner. Explain to your partner how to convert a ratio that does not compare a number to 100 as a percent. Then have your partner explain to you how to change from a percent to a fraction in simplest form. Both of you should use examples as well as general explanations.

See students' work.
A ratio is a comparison of two numbers by division. When a ratio compares a number to 100, it can be written as a percent. To write a ratio or fraction as a percent, find an equivalent fraction with a denominator of 100. You can also use the meaning of percent to change percents to fractions.

Example 1
Write \( \frac{19}{35} \) as a percent.

Since \( 100 \div 35 = 2.857 \), multiply the numerator and denominator by 5.

Example 2
Write 92% as a fraction in simplest form.

Definition of percent

Simplify.

Exercises

Write each ratio as a percent.

1. \( \frac{14}{100} \) 14%

2. \( \frac{27}{100} \) 27%

3. \( \frac{34.5}{100} \) 34.5%

4. 18 per 100 18%

5. 21:100 21%

6. 96:100 96%

Write each fraction as a percent.

7. \( \frac{3}{100} \) 3%

8. \( \frac{14}{100} \) 14%

9. \( \frac{2}{5} \) 40%

10. \( \frac{1}{20} \) 5%

11. \( \frac{13}{25} \) 52%

12. \( \frac{4}{10} \) 40%

Write each percent as a fraction in simplest form.

13. 35% \( \frac{7}{20} \)

14. 18% \( \frac{9}{50} \)

15. 75% \( \frac{3}{4} \)

16. 80% \( \frac{4}{5} \)

17. 16% \( \frac{4}{25} \)

18. 15% \( \frac{3}{20} \)

Chapter 4

<table>
<thead>
<tr>
<th>Lesson 4-6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Skills Practice</strong></td>
</tr>
<tr>
<td><strong>Fractions and Percents</strong></td>
</tr>
</tbody>
</table>

Write each ratio as a percent.

1. 26 out of 100 26%

2. 2.5 per 100 5%

3. 13:100 13%

4. \( \frac{39}{100} \) 39%

5. 12.5 per 100 12.5%

6. 51 out of 100 51%

Write each fraction as a percent.

7. \( \frac{7}{10} \) 70%

8. \( \frac{6}{50} \) 12%

9. \( \frac{13}{20} \) 65%

10. \( \frac{30}{50} \) 60%

11. \( \frac{7}{20} \) 35%

12. \( \frac{12}{20} \) 60%

13. \( \frac{23}{25} \) 92%

14. \( \frac{3}{10} \) 30%

15. \( \frac{17}{50} \) 34%

Write each percent as a fraction in simplest form.

16. 15% \( \frac{3}{20} \)

17. 85% \( \frac{17}{20} \)

18. 1% \( \frac{1}{100} \)

19. 70% \( \frac{7}{10} \)

20. 25% \( \frac{1}{4} \)

21. 19% \( \frac{19}{100} \)

22. 83% \( \frac{33}{100} \)

23. 22% \( \frac{1}{5} \)

24. 95% \( \frac{19}{20} \)
Write each ratio as a percent.

1. Three out of every 10 students in Mr. Chan’s class bring their lunch to school. Write this ratio as a percent.
   \[
   \frac{3}{10} = 30\% 
   \]

2. In 2000, 57 out of every 100 school age children (ages 6 to 17 years) had access to a computer both at home and at school. Write this ratio as a percent.
   \[
   \frac{57}{100} = 57\% 
   \]

3. In one town, the sales tax is 8%. Write this percent as a fraction in simplest form.
   \[
   0.08 = \frac{8}{100} = \frac{2}{25} 
   \]

4. Ms. Agosto surveyed her class and found that 15 out of 30 students brushed their teeth more than twice a day. What percent of students brushed more than twice a day?
   \[
   \frac{15}{30} = 50\% 
   \]

5. A local retail store was having a sale and offered all their merchandise at a 25% discount. Write this percent as a fraction in simplest form.
   \[
   0.25 = \frac{25}{100} = \frac{1}{4} 
   \]

6. About 64% of all individuals who have flown in space from 1961 to 2001 are from the United States. Write this percent as a fraction in simplest form.
   \[
   0.64 = \frac{64}{100} = \frac{16}{25} 
   \]
Get Ready for the Lesson

Read the introduction at the top of page 206 in your textbook. Write your answers below.

1. Write the percent of students who read for fun as a fraction.

2. Write the fraction as a decimal.

3. Compare the decimal in Question 2 with its percent form. Identify any similarities or differences.

Read the Lesson

4. Describe each step in changing a percent to a decimal. Sample answer: 1) Write the percent as a fraction with the denominator equal to 100. 2) Write the fraction as a decimal, moving the decimal point two places to the left.

5. Describe each step in changing a percent to a decimal by first writing the percent as a fraction. Sample answer: 1) Write the percent as a fraction with the denominator equal to 100. 2) If necessary, multiply the numerator and denominator by a power of 10 to remove a decimal in the numerator. Then simplify. 3) Write the fraction as a decimal.

6. Describe how to write a percent as a decimal without writing the percent as a fraction. Sample answer: 1) Write any fractions as decimals. 2) Divide by 100 (move the decimal point two places to the left). 3) Remove the % symbol.

Remember What You Learned

7. Work with a partner. Think of a way that will help you remember which way to move the decimal when you go from a percent to a decimal and which way to move it when you go from a decimal to a percent. See students' work.
### Answers (Lesson 4-7)

#### Study Guide and Intervention

**Percents and Decimals**

To write a percent as a decimal, divide the percent by 100 and remove the percent symbol. To write a decimal as a percent, multiply the decimal by 100 and add the percent symbol.

#### Exercises

1. Write each percent as a decimal.
   1. 5% = 0.05
   2. 35% = 0.35
   3. 47% = 0.47
   4. 84% = 0.84
   5. 91% = 0.91
   6. 14% = 0.14
   7. 12% = 0.12
   8. 11% = 0.11
   9. 3% = 0.03
   10. 4% = 0.04

2. Write each decimal as a percent.
   1. 0.06 = 6%
   2. 0.25 = 25%
   3. 0.5 = 50%
   4. 0.625 = 62.5%
   5. 0.75 = 75%
   6. 0.91 = 91%
   7. 0.1 = 10%
   8. 0.04 = 4%
   9. 0.075 = 7.5%
   10. 0.142 = 14.2%

### Chapter 4

Glencoe California Mathematics, Grade 6

A20
**4-7 Practice**

**Percents and Decimals**

Write each percent as a decimal.

1. 35\% \rightarrow 0.35
2. 90\% \rightarrow 0.90
3. 5\% \rightarrow 0.05
4. 1\% \rightarrow 0.01
5. 21.8\% \rightarrow 0.218
6. 64.8\% \rightarrow 0.648
7. 4.1\% \rightarrow 0.041
8. 8.5\% \rightarrow 0.085
9. \frac{39}{50} \rightarrow 0.39 \rightarrow \frac{39}{50}
10. 0.174 \rightarrow 0.174
11. \frac{40}{3} \rightarrow 0.40 \rightarrow \frac{40}{3}
12. \frac{88}{5} \rightarrow 0.88 \rightarrow \frac{88}{5}

Write each decimal as a percent.

13. 0.4 \rightarrow 40\%
14. 0.8 \rightarrow 80\%
15. 3.7 \rightarrow 370\%
16. 9.1 \rightarrow 910\%
17. 0.07 \rightarrow 7\%
18. 0.03 \rightarrow 3\%
19. 0.25 \rightarrow 25\%
20. 0.59 \rightarrow 59\%
21. 0.375 \rightarrow 37.5\%
22. 0.123 \rightarrow 12.3\%
23. 0.005 \rightarrow 0.5\%
24. 0.6019 \rightarrow 60.19\%

Replace each \( \div \) with >, <, or = to make a true sentence.

25. 1.5 \div 15\% >
26. 0.86 \div 8.8\% >
27. 0.33 \div 0.33 =

28. 90\% \div 0.09 >
29. 0.26 \div 27\% <
30. 65.4\% \div 0.645 >

**ANALYZE TABLES**

For Exercises 31-33, use the table and the information given.

The table lists the approximate milk fat content of 5 types of milk products.

<table>
<thead>
<tr>
<th>Milk Product</th>
<th>Percent Milk Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy Cream</td>
<td>36.7%</td>
</tr>
<tr>
<td>Light Cream</td>
<td>19.2%</td>
</tr>
<tr>
<td>Whole Milk</td>
<td>3.5%</td>
</tr>
<tr>
<td>Low-Fat Milk</td>
<td>1.5%</td>
</tr>
<tr>
<td>Skim Milk</td>
<td>0.05%</td>
</tr>
</tbody>
</table>

31. Which product has the highest milk fat content?
   \( \text{heavy cream} \)

32. Find the approximate number of grams of milk fat in a 200-gram serving of whole milk.
   \( \text{7 grams} \)

33. Which milk product will have approximately 15.56 grams of milk fat in an 80-gram serving?
   \( \text{light cream} \)

**Word Problem Practice**

**Percents and Decimals**

1. AREA: New Mexico’s land area is about 0.03 of the total area of the United States. What percent is New Mexico’s land area of the total area of the United States? \( \text{3%} \)

2. SCALE MODEL: A scale model of a building is 0.25 the actual size. What percent of the actual size of the building is the model? \( \text{25%} \)

3. NFL COACHES: Don Shula ranks among the most successful coaches in the National Football League. In his career, he won 0.665 of his games. Write the decimal as a percent. \( \text{66.5%} \)

4. SOFTBALL: Jenny’s batting average is 0.346. Write the decimal as a percent. \( \text{34.6%} \)

5. VITAMINS: A multiple vitamin contains 450 milligrams of calcium. This is 45% of the recommended daily allowance. Write the percent as a decimal. \( \text{0.45} \)

6. BASKETBALL: Tao makes 74% of his free throws. Write the percent as a decimal. \( \text{0.74} \)

7. SALES TAX: The sales tax in a town is 7.25%. Write the percent as a decimal. \( \text{0.0725} \)

8. FIELD TRIP: In Ms. Silver’s English class, \( \frac{20}{4} \) of the students signed up to visit a local museum. Write the percent as a decimal. \( \text{0.2025} \)
Lesson 4–8

Get Ready for the Lesson

Complete the Mini Lab at the top of page 211 in your textbook. Write your answers below.

1. Add a second floor to each building. Record the total number of cubes used in a table like the one shown below.

<table>
<thead>
<tr>
<th>Number of Floors</th>
<th>Building 1</th>
<th>Building 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

2. Add floors until each building has five floors.

3. Describe two buildings that have the same number of cubes. Both buildings have the same number of cubes when Building 1 has 4 floors and Building 2 has 3 floors.

4. If you keep adding floors, will the two buildings have the same number of cubes again? Explain. Yes; Sample answer: Every fourth floor in Building 1 will have the same number of cubes as every third floor in Building 2.

Read the Lesson

5. What is a least common multiple of two or more numbers?

It is the least of their common multiples excluding zero.

6. Describe, in your own words, the first method used to find the LCM in Example 1 at the bottom of page 259. Method one requires listing several multiples of 6 and 10 and looking for the least common of the multiples.

Remember What You Learned

7. Explain how to find the LCM of two or more numbers when you know the prime factorization of each number. Give an example.

Sample answer: The LCM is the product of the greatest powers of each of the prime factors; to find the LCM of 16 and 18, first find the prime factorization of each number:

- 16 = 2^4
- 18 = 2 × 3^2

Then the LCM is the product of the greatest power of 2 and the greatest power of 3, or 2^4 × 3^2 = 144.

Enrichment

African-American Scientists and Inventors

When you buy a pair of shoes, you usually have a wide variety of styles, sizes, and prices to choose from. It is the work of an African-American inventor, Jan Matzeliger (1852–1889), that makes this possible. In 1882, Matzeliger patented a lasting machine that could shape the upper portion of a shoe and attach it to the sole in a fraction of the time it took to do the job by hand. Using this machine, shoe manufacturers were able to increase production and reduce prices dramatically.

African Americans have made many significant contributions to mathematics, science, and invention. By solving the percent problems and matching the problem and the correct solution, you will learn more about just a few of them.

Solutions

A. 20 Benjamin Banneker
B. 21 Majorie Lee Browne
C. 18 Lewis Latimer
D. 17.5 Jane Cooke Wright

1. 35% of 50 is what number?

This physician researched and tested chemotherapy as a method of treating cancer. In 1952, she became head of the Cancer Research Foundation at Harlem Hospital.

2. What percent of 75 is 15?

This mathematician was part of the team of surveyors who created the street plan for Washington, D.C. in the late eighteenth century.

3. 4.5% of 400 is what number?

In 1876, this engineer drew up the plans that accompanied Alexander Graham Bell’s application for a patent on the telephone.

4. 120% of what number is 25.2?

In 1949, she became one of the first two African-American women to earn a doctorate in mathematics. She was head of the mathematics department at North Carolina Central University from 1951 to 1970.

Sample answer: The LCM is the product of the greatest powers of each of the prime factors; to find the LCM of 16 and 18, first find the prime factorization of each number:

- 16 = 2^4
- 18 = 2 × 3^2

Then the LCM is the product of the greatest power of 2 and the greatest power of 3, or 2^4 × 3^2 = 144.
Answers (Lesson 4-8)

Skills Practice

Least Common Multiple

Find the LCM of each set of numbers.

1. 2, 8
   - Answer: 8

2. 6, 10
   - Answer: 30

3. 10, 11
   - Answer: 110

4. 10, 12
   - Answer: 60

5. 9, 18
   - Answer: 18

6. 4, 22
   - Answer: 44

7. 12, 30
   - Answer: 60

8. 4, 13
   - Answer: 52

9. 25, 30
   - Answer: 150

10. 250, 30
    - Answer: 750

11. 200, 18
    - Answer: 1,800

12. 70, 90
    - Answer: 630

13. 18, 54
    - Answer: 54

14. 30, 65
    - Answer: 390

15. 180, 252
    - Answer: 1,260

16. 20, 55
    - Answer: 220

17. 21, 60
    - Answer: 420

18. 3, 5, 10
    - Answer: 30

19. 3, 4, 13
    - Answer: 156

20. 4, 10, 12
    - Answer: 60

21. 6, 15, 20
    - Answer: 60

22. 48, 16, 3
    - Answer: 48

23. 66, 55, 44
    - Answer: 660

24. 29, 58, 4
    - Answer: 116

Study Guide and Intervention

Least Common Multiple

Example 1

Find the LCM of 15 and 20 by listing multiples.

List the multiples.

- Multiples of 15: 15, 30, 45, 60, ...
- Multiples of 20: 20, 40, 60, ...

Notice that 60, 120, ... are common multiples. So, the LCM of 15 and 20 is 60.

Example 2

Find the LCM of 8 and 12 using prime factors.

Write the prime factorization.

- 8 = 2 × 2 × 2
- 12 = 2 × 2 × 3

The prime factors of 8 and 12 are 2 and 3.

Multiply the greatest power of each prime factor.

- 2^3 × 3
- LCM = 24

Exercises

6.NS.2.4

Find the LCM of each set of numbers.

1. 2, 8
   - Answer: 8

2. 6, 10
   - Answer: 30

3. 10, 11
   - Answer: 110

4. 10, 12
   - Answer: 60

5. 9, 18
   - Answer: 18

6. 4, 22
   - Answer: 44

7. 12, 30
   - Answer: 60

8. 4, 13
   - Answer: 52

9. 25, 30
   - Answer: 150

10. 250, 30
    - Answer: 750

11. 200, 18
    - Answer: 1,800

12. 70, 90
    - Answer: 630

13. 18, 54
    - Answer: 54

14. 30, 65
    - Answer: 390

15. 180, 252
    - Answer: 1,260

16. 20, 55
    - Answer: 220

17. 21, 60
    - Answer: 420

18. 3, 5, 10
    - Answer: 30

19. 3, 4, 13
    - Answer: 156

20. 4, 10, 12
    - Answer: 60

21. 6, 15, 20
    - Answer: 60

22. 48, 16, 3
    - Answer: 48

23. 66, 55, 44
    - Answer: 660

24. 29, 58, 4
    - Answer: 116

A multiple of a number is the product of that number and any whole number. The least nonzero multiple of two or more numbers is the least common multiple (LCM) of the numbers.

The least common multiple of 15 and 20 is 60.

For 8 and 12, the prime factors are 2 and 3.

The LCM is 2^3 × 3 = 24.
1. **PROMOTION** In a promotion for a local delicatessen, every eighth customer will get a free sandwich and every sixth customer will get a free drink. Which customer will be first to get both a free sandwich and a free drink? 24th customer

2. **WORK** Alano and Abey both work at night. Alano has every fourth night off and Abey has every sixth night off. If they are both off tonight, how long will it be before they are both off again? 12 nights

3. **RADIO** A radio station is giving away a discount coupon to every twelfth caller and a free concert ticket to every twentieth caller. Which caller will be first to win both the coupon and the ticket? 60th caller

4. **MUSIC** Faith spent the same amount of money on cassette tapes as she did on CDs. If tapes cost $12 and CDs cost $16, what is the least amount of money she could have spent on each? $48

5. **BIKES** Three bike riders ride around a circular path. The first rider circles the path in 12 minutes, the second in 18 minutes, and the third in 24 minutes. If they all start at the same place, at the same time, and go in the same direction, after how many minutes will they meet at the starting point? 72 min

6. **PAPER GOODS** At a party store, paper cups come in packages of 15, paper plates come in packages of 30, and napkins come in packages of 20. In order to have the same number of cups, plates, and napkins, what is the least number of each that must be purchased? 60

---

**Find the LCM of each set of numbers.**

- 1. 8, 12: 24
- 2. 10, 25: 50
- 3. 12, 18: 36
- 4. 20, 30: 60
- 5. 5, 8, 9: 72
- 6. 15, 35: 105
- 7. 3, 5, 7: 105
- 8. 4, 10, 12: 60
- 9. 9, 12, 15: 180
- 10. 5, 15, 20: 60
- 11. 14, 21, 42: 42
- 12. 15, 18, 30: 90

**Write two numbers whose LCM is the given number.**

- 13. 2 feet, 1 yard: 2 yards or 6 feet
- 14. 6¢, 18¢: 24¢
- 15. 40 seconds, 1 minute: 2 minutes
- 16. 24: Sample answer: 3 and 8
- 17. 63: Sample answer: 7 and 9
- 18. 50: Sample answer: 2 and 25

19. **SECURITY** In a large industrial complex, three security teams work different types of security checks. The first team makes a complete round in 3 hours, the second team makes a complete round in 2 hours, while the third team makes a complete round in 4 hours. If all three teams start security checks at 7 A.M., when will be the next time all three teams finish a security check at the same time? 7 P.M.

20. **COOKIES** A recipe for large oatmeal cookies will make 15 cookies. A recipe for chocolate chip cookies will make 2 dozen cookies. If you want to have the same number of each type of cookie, what is the least number of each that you will need to make using complete recipes? 120 cookies

21. **ICE SKATING** Three friends ice skate at different speeds. Parcel skates one lap in 45 seconds. It takes Hunsel 1 1/2 minutes to skate one lap and Forrest takes only 30 seconds to skate a lap. If they started out together, in how many minutes will they meet next? 1 1/2 minutes
Periodic Cicadas

Cicadas, also commonly known as locusts, are insects that inhabit much of the eastern United States. Some cicadas are called periodic because they have life cycles that span periods of several years. The Magicicada is a kind of cicada that has an unusually long life cycle of 13 or 17 years. These 13-year and 17-year cicadas spend much of their lives living underground. After 12 or 16 years, the cicadas start to burrow upward. All at once they emerge from the ground, taking flight and eating most of the leaves on nearby plants.

While scientists do not know for sure why the Magicicada life cycles last for 13 or 17 years, they do have several theories. One theory is that this life cycle pattern makes it easier for the cicadas to find food. Another theory is that the pattern helps the cicada avoid predators.

For Exercises 1–4, refer to the following information.

Suppose there are many kinds of cicadas. For each of the pair below, find out how many years would pass before each would again emerge at the same time.

1. 11-year and 15-year cicadas
   165 yr
2. 12-year and 16-year cicadas
   48 yr
3. 14-year and 18-year cicadas
   126 yr
4. 13-year and 17-year cicadas
   221 yr

5. In what year will the 13-year and 17-year cicadas that emerged in the summer of 1998 once again emerge at the same time? Explain.
   221 yr; Sample answer: since the LCM of 13 and 17 is 221, they emerge together every 221 years.

For Exercises 6–8, refer to the following information.

Suppose the cicada has three predators that have life cycles of 2, 3, and 5 years. The population of cicadas was nearly wiped out one summer because all three predators emerged when the cicadas emerged.

6. After how many years will the three predators emerge together again? 30 yr
7. For each of the cicada populations in Exercises 1–4, find the number of years before the cicada population would again emerge at the same time as all three predators. Record your answers in the table below.

<table>
<thead>
<tr>
<th>Cicada Type</th>
<th>11-year</th>
<th>12-year</th>
<th>13-year</th>
<th>14-year</th>
<th>15-year</th>
<th>16-year</th>
<th>17-year</th>
<th>18-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years</td>
<td>330</td>
<td>60</td>
<td>390</td>
<td>210</td>
<td>120</td>
<td>240</td>
<td>510</td>
<td>90</td>
</tr>
</tbody>
</table>

8. Which two cicada populations have the best chance of survival? Explain.
   Sample answer: The 13-year and the 17-year cicadas, because they have the longest period of time before they have to face all three predators at once.
Lesson Reading Guide
Comparing and Ordering Rational Numbers

Get Ready for the Lesson
Complete the Mini Lab at the top of page 215 in your textbook.

Write your answers below.

1. \( \frac{7}{8} \) \( \frac{3}{8} \) \( \frac{2}{3} \) \( \frac{1}{3} \)
2. \( \frac{5}{8} - \frac{1}{8} \) \( \frac{1}{3} \) \( \frac{1}{2} \) \( \frac{1}{4} \)
3. \( \frac{13}{8} - \frac{3}{8} \) \( \frac{13}{8} \) \( \frac{13}{8} \) \( \frac{13}{8} \)
4. \( -\frac{15}{8} - \frac{15}{8} \) \( \frac{15}{8} \) \( \frac{15}{8} \) \( \frac{15}{8} \)
5. \( \frac{1}{2} - \frac{3}{4} \) \( \frac{1}{2} \) \( \frac{1}{2} \) \( \frac{1}{2} \)
6. \( \frac{1}{2} - \frac{1}{4} \) \( \frac{1}{4} \) \( \frac{1}{4} \) \( \frac{1}{4} \)

7. MAKE A CONJECTURE Which number is less: \( \frac{3}{4} \) or \( \frac{2}{3} \)? Sample answer: Use a number line to explain your reasoning.

Read the Lesson

8. What are two ways in which you can compare fractions? 1) Rename the fractions using the LCD. 2) Write each fraction as a decimal and then compare the decimals. 3) Estimate, using a number line if necessary.

9. Complete the table of common fraction-decimal-percent equivalents.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>0.5</td>
<td>50%</td>
</tr>
<tr>
<td>( \frac{1}{4} )</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>( \frac{1}{3} )</td>
<td>0.333</td>
<td>33%</td>
</tr>
<tr>
<td>( \frac{1}{10} )</td>
<td>0.1</td>
<td>10%</td>
</tr>
</tbody>
</table>

Find the LCD of each pair of fractions.

1. \( \frac{1}{2} \) \( \frac{3}{4} \)
2. \( \frac{3}{4} \) \( \frac{7}{8} \)
3. \( \frac{3}{4} \) \( \frac{7}{10} \)

Replace each \( \ast \) with \(<\), \(>,\) or \(=\) to make a true sentence.

4. \( \frac{3}{4} \ast \frac{9}{12} \)
5. \( \frac{5}{8} \ast \frac{10}{12} \)
6. \( \frac{3}{4} \ast \frac{7}{8} \)
7. \( \frac{5}{8} \ast \frac{6}{9} \)
8. \( \frac{8}{12} \ast \frac{10}{15} \)
9. \( \frac{5}{8} \ast \frac{6}{11} \)
10. \( \frac{3}{17} \ast \frac{1}{2} \)
11. \( \frac{9}{10} \ast \frac{17}{19} \)

Remember What You Learned

10. How are the following sets of numbers related: whole numbers, rational numbers, integers? The set of rational numbers includes all of the integers and whole numbers. The set of integers includes all whole numbers.

11. In this lesson you learned about the LCD. What do each of the following abbreviations stand for: LCD, LCM, and GCF? Sample answer: LCD: least common denominator; LCM: least common multiple; GCF: greatest common factor. The LCD of a set of fractions is the LCM of the denominators of the fractions.
### 4-9 Skills Practice

#### Comparing and Ordering Rational Numbers

Find the LCD of each pair of fractions.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\frac{4}{5})</td>
<td>(\frac{3}{5})</td>
</tr>
<tr>
<td>2.</td>
<td>(\frac{5}{12})</td>
<td>(\frac{7}{24})</td>
</tr>
<tr>
<td>3.</td>
<td>(\frac{6}{25})</td>
<td>(\frac{3}{7})</td>
</tr>
<tr>
<td>4.</td>
<td>(\frac{7}{15})</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>5.</td>
<td>(\frac{7}{11})</td>
<td>(\frac{3}{5})</td>
</tr>
<tr>
<td>6.</td>
<td>(\frac{5}{17})</td>
<td>(\frac{7}{8})</td>
</tr>
<tr>
<td>7.</td>
<td>(\frac{5}{12})</td>
<td>(\frac{7}{10})</td>
</tr>
<tr>
<td>8.</td>
<td>(\frac{15}{16})</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>9.</td>
<td>(\frac{5}{8})</td>
<td>(\frac{3}{5})</td>
</tr>
</tbody>
</table>

Replace each @ with <, >, or = to make a true sentence.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td>(\frac{3}{10})</td>
<td>(\frac{2}{3})</td>
</tr>
<tr>
<td>11.</td>
<td>(\frac{3}{7})</td>
<td>(\frac{5}{11})</td>
</tr>
<tr>
<td>12.</td>
<td>(\frac{9}{12})</td>
<td>(\frac{3}{4})</td>
</tr>
<tr>
<td>13.</td>
<td>(\frac{12}{13})</td>
<td>(\frac{4}{15})</td>
</tr>
<tr>
<td>14.</td>
<td>(\frac{4}{5})</td>
<td>(\frac{5}{4})</td>
</tr>
<tr>
<td>15.</td>
<td>(\frac{17}{30})</td>
<td>(\frac{33}{20})</td>
</tr>
<tr>
<td>16.</td>
<td>(\frac{35}{60})</td>
<td>(\frac{39}{84})</td>
</tr>
<tr>
<td>17.</td>
<td>(\frac{3}{11})</td>
<td>(\frac{3}{7})</td>
</tr>
<tr>
<td>18.</td>
<td>(\frac{2}{3})</td>
<td>(\frac{9}{7})</td>
</tr>
</tbody>
</table>

Order each set of ratios from least to greatest.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>19.</td>
<td>0.46, 0.46, (\frac{9}{20})</td>
<td>0.20</td>
</tr>
<tr>
<td>21.</td>
<td>(\frac{9}{20}), 0.46, 0.48</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Determine whether each number is rational. Write yes or no.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>22.</td>
<td>2.3232323232...</td>
</tr>
<tr>
<td>23.</td>
<td>(\frac{7}{19})</td>
</tr>
<tr>
<td>24.</td>
<td>4.3</td>
</tr>
</tbody>
</table>

### 4-9 Practice

#### Comparing and Ordering Rational Numbers

Replace each @ with <, >, or = to make a true sentence.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\frac{5}{6}) @ (\frac{1}{3})</td>
<td>&gt;</td>
</tr>
<tr>
<td>2.</td>
<td>(\frac{5}{8}) @ (\frac{9}{10})</td>
<td>&lt;</td>
</tr>
<tr>
<td>3.</td>
<td>(\frac{6}{9}) @ (\frac{4}{6})</td>
<td>=</td>
</tr>
<tr>
<td>4.</td>
<td>(\frac{2}{7}) @ (\frac{1}{8})</td>
<td>&gt;</td>
</tr>
<tr>
<td>5.</td>
<td>(\frac{15}{21}) @ (\frac{12}{18})</td>
<td>&gt;</td>
</tr>
<tr>
<td>6.</td>
<td>(\frac{24}{32}) @ (\frac{36}{48})</td>
<td>=</td>
</tr>
<tr>
<td>7.</td>
<td>(\frac{8}{11}) @ (\frac{10}{13})</td>
<td>&lt;</td>
</tr>
<tr>
<td>8.</td>
<td>(\frac{14}{15}) @ (\frac{19}{20})</td>
<td>&lt;</td>
</tr>
<tr>
<td>9.</td>
<td>(\frac{1}{5}) @ (\frac{4}{10})</td>
<td>=</td>
</tr>
<tr>
<td>10.</td>
<td>(\frac{7}{9}) @ (\frac{2}{3})</td>
<td>&lt;</td>
</tr>
<tr>
<td>11.</td>
<td>(\frac{17}{20}) @ (\frac{1}{8})</td>
<td>&gt;</td>
</tr>
<tr>
<td>12.</td>
<td>(\frac{9}{2}) @ (\frac{5}{6})</td>
<td>&gt;</td>
</tr>
</tbody>
</table>

Determine whether each number is rational. Write yes or no. Explain your reasoning.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13.</td>
<td>(\frac{3}{4})</td>
</tr>
<tr>
<td>14.</td>
<td>0.65</td>
</tr>
<tr>
<td>15.</td>
<td>0.42857...</td>
</tr>
<tr>
<td>16.</td>
<td>(\frac{7}{10})</td>
</tr>
<tr>
<td>17.</td>
<td>(\frac{3}{5})</td>
</tr>
</tbody>
</table>

Order each set of numbers from least to greatest.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>22.</td>
<td>63%</td>
<td>0.65</td>
</tr>
<tr>
<td>23.</td>
<td>(\frac{7}{8})</td>
<td>0.98, 0.85%</td>
</tr>
<tr>
<td>24.</td>
<td>0.42, (\frac{1}{12})</td>
<td>0.5</td>
</tr>
<tr>
<td>25.</td>
<td>0.65</td>
<td>0.42</td>
</tr>
<tr>
<td>26.</td>
<td>0.8</td>
<td>0.85%</td>
</tr>
<tr>
<td>27.</td>
<td>2%</td>
<td>0.02</td>
</tr>
</tbody>
</table>

25. **BASEBALL** The pitchers for the home team had 12 strikeouts for 32 batters, while the pitchers for the visiting team had 15 strikeouts for 35 batters. Which pitching team had a greater fraction of strikeouts?

26. **TRANSPORTATION** To get to school, 38% of the students ride in the family vehicle, 5 out of 12 students ride on the school bus, and 0.12 of the students ride a bike. Order the types of transportation students use to get to school from least to greatest.

- 0.12 (riding a bike)
- 38% (riding in the family vehicle)
- 5 out of 12 (riding the school bus)
1. RAIN The amount of rainfall was measured after a recent storm. The north side of town received \( \frac{3}{4} \) inch of rain, and the south side received \( \frac{13}{15} \) inch of rain. Which side of town received more rain from the storm? **North**

2. MOVIES Because he sees movies at his local theater so often, Delmar is being offered a discount. He can have either \( \frac{1}{3} \) off his next ticket or 30% off his next ticket. Which discount should Delmar choose? Explain. \( \frac{1}{3} \) off; \( \frac{1}{3} > 30\%\)

3. TRACK Willie runs the 110-meter hurdles in \( \frac{7}{12} \) seconds, and Anier runs it in \( \frac{7}{11} \) seconds. Which runner is faster? **Anier**

4. FARMING Cassie successfully harvested \( \frac{7}{12} \) of her crop, and Robert successfully harvested 58% of his crop. Which person successfully harvested the larger portion of his or her crop? **Cassie**

5. TRANSPORTATION My-Lien has enough room in her truck to move 3.385 tons of gravel. Her father has asked her to move 3 \( \frac{5}{16} \) tons. Will My-Lien be able to move all of the gravel in only one trip? Explain. **Yes;** \( \frac{5}{16} < 3.385\)

6. WOOD WORKING Kashi has a bolt that is \( \frac{4}{3} \) inch wide, and she drilled a hole 0.6 inch wide. Is the hole large enough to fit the bolt? Explain. **No;** \( \frac{5}{8} > 0.6\)

7. PIZZA In a recent pizza-eating contest, Alfonso ate \( \frac{3}{4} \) pizzas, Della ate \( \frac{1}{10} \) pizzas, and Delesn ate \( \frac{9}{4} \) pizzas. Which person won the contest? **Delesn**

8. STUDYING For a recent algebra exam, Pat studied 1.25 hours, Toni studied 1.16 hours, and Morgan studied 1.9 hours. List the students in order by who studied the most. **Morgan, Toni, Pat**

**Enrichment**

Intersection and Union of Sets

The darker shaded areas in the Venn diagrams show the union and intersection of sets \( A \) and \( B \).

For example, if \( A = \{1, 2, 3, 4\} \) and \( B = \{3, 4, 5, 6\} \), then their union and intersection are written as:

- Union: \( A \cup B = \{1, 2, 3, 4, 5, 6\} \)
- Intersection: \( A \cap B = \{3, 4\} \)

Draw a Venn diagram for sets \( A \) and \( B \). Then write the numbers included in \( A \cup B \) and \( A \cap B \). In Exercises 2 and 4, record the numbers as decimals.

1. \( A = \{\text{integers between 0 and 77}\} \)
   - \( B = \{\text{factors of 12}\} \)
   - \( A \cup B = \{1, 2, 3, 4, 5, 6, 12\} \)
   - \( A \cap B = \{1, 2, 3, 4, 6\} \)

2. \( A = \{\text{one-place decimals between 0 and 0.5}\} \)
   - \( B = \{\text{fractions with 1, 2, 3, or 4 as numerator and 5 as a denominator}\} \)
   - \( A \cup B = \{0.1, 0.2, 0.3, 0.4, 0.6, 0.8\} \)
   - \( A \cap B = \{0.2, 0.4\} \)

3. \( A = \{\text{perfect squares between 0 and 30}\} \)
   - \( B = \{\text{odd whole numbers less than 10}\} \)
   - \( A \cup B = \{1, 3, 4, 5, 7, 9, 16, 25\} \)
   - \( A \cap B = \{1, 9\} \)

4. \( A = \{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\} \)
   - \( B = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\} \)
   - \( A \cup B = \{0.1, 0.2, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\} \)
   - \( A \cap B = \{0.3\} \)
Chapter 4 Assessment Answer Key

Quiz 1 (Lessons 4-1 through 4-3) Page 67
1. \(2^2 \times 7\)
2. \(2^2 \times 11\)
3. \(3^2 \times 5 \times 7\)
4. \(2^2 \times 5^3\)
5. \(2 \times 2 \times 2 \times 3 \times a \times b\)
6. \(2 \times 2 \times 3 \times x \times y \times y\)
7. 15
8. 2
9. 9w
10. 5r

Quiz 2 (Lessons 4-4 and 4-5) Page 67
1. B
2. \(\frac{3}{16}\)
3. \(\frac{5}{6}\)
4. 2.625
5. 0.7

Quiz 3 (Lessons 4-6 and 4-7) Page 68
1. \(\frac{33}{50}\)
2. 35%
3. 38%
4. 17%
5. 0.84
6. 0.06
7. 0.255
8. 28%
9. 29.9%
10. 4%

Quiz 4 (Lessons 4-8 and 4-9) Page 68
1. B
2. 72
3. 900
4. <
5. =

Mid-Chapter Test Page 69
1. B
2. J
3. B
4. G
5. B
6. G
7. C
8. composite
9. \(\frac{2 \times 2 \times 5 \times 5 \times m \times n}{m \times n}\)
10. 15w
11. \(\frac{4}{3}\)
12. \(\frac{3}{4}\)
13. 2.124
14. 7.875
Chapter 4 Assessment Answer Key

Vocabulary Test
Page 70

1. prime factorization
2. equivalent fractions
3. terminating decimal
4. greatest common factor
5. multiple
6. Rational numbers
7. ratio
8. composite number
9. Venn diagram
10. 100

11. Sample answer: the form of a fraction in which the GCF of the numerator and denominator is 1
12. a ratio that compares a number to 100

Form 1
Page 71

1. B
2. J
3. B
4. J
5. A
6. J
7. C
8. H
9. B
10. F
11. C
12. F
13. D
14. G
15. A
16. H
17. B
18. F
19. B
20. H

B: 24.146
# Chapter 4 Assessment Answer Key

<table>
<thead>
<tr>
<th>Form 2A</th>
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<th>Form 2B</th>
<th>Page 75</th>
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<tbody>
<tr>
<td>1. <em>B</em></td>
<td>12. <em>F</em></td>
<td>1. <em>C</em></td>
<td>12. <em>H</em></td>
</tr>
<tr>
<td>2. <em>H</em></td>
<td>13. <em>C</em></td>
<td>2. <em>F</em></td>
<td>13. <em>D</em></td>
</tr>
<tr>
<td>4. <em>G</em></td>
<td>15. <em>D</em></td>
<td>4. <em>G</em></td>
<td>15. <em>B</em></td>
</tr>
<tr>
<td>5. <em>A</em></td>
<td>16. <em>J</em></td>
<td>5. <em>C</em></td>
<td>16. <em>H</em></td>
</tr>
<tr>
<td>6. <em>H</em></td>
<td>17. <em>B</em></td>
<td>6. <em>H</em></td>
<td>17. <em>B</em></td>
</tr>
<tr>
<td>7. <em>C</em></td>
<td>18. <em>F</em></td>
<td>7. <em>B</em></td>
<td>18. <em>F</em></td>
</tr>
<tr>
<td>10. <em>G</em></td>
<td></td>
<td>10. <em>G</em></td>
<td></td>
</tr>
<tr>
<td>11. <em>C</em></td>
<td></td>
<td>11. <em>C</em></td>
<td></td>
</tr>
</tbody>
</table>

B: **50%**

B: **5%**
Chapter 4 Assessment Answer Key

Form 2C
Page 77

1. 3, 5, 9; 3 and 5 prime, 9 composite

2. $2 \times 3 \times 17$

3. $2 \times 3^3 \times 7$

4. 21

5. 12

6. $6a$

7. $\frac{3}{4}$

8. $\frac{1}{3}$

9. 0.8

10. 6.75

11. 0.78

12. $\frac{1}{50}$

13. $\frac{17}{25}$

Page 78

14. 36.1%

15. 19%

16. 80%

17. 90

18. 105

19. 255

20. 51

21. >

22. >

23. <

24. $\frac{1}{4}$ in., $\frac{10}{25}$ in., 0.5 in.

25. $\frac{7}{13}$; 0.538

B: 3
Chapter 4 Assessment Answer Key

Form 2D
Page 79

1. 4 and 9 composite

2. \(2^3 \times 3^2\)

3. \(2 \times 7^2\)

4. 8

5. 4

6. \(2z\)

7. \(\frac{7}{12}\)

8. \(\frac{21}{34}\)

9. 0.27

10. 7.6

11. 0.84

12. \(\frac{3}{50}\)

13. \(\frac{22}{25}\)

Page 80

14. 73.2%

15. 27%

16. 52%

17. 360

18. 288

19. 110

20. 50

21. <

22. >

23. =

24. 0.8 ft, \(\frac{7}{8}\) ft, \(\frac{15}{16}\) ft

25. \(\frac{5}{12}\); 0.417

B: 5
Chapter 4 Assessment Answer Key

Form 3
Page 81

1. 10 composite

2. \(3 \times 17\)

3. \(2^3 \times 5 \times 13\)

4. \(7\)

5. \(6\)

6. \(4b\)

7. \(\frac{2}{9}\)

8. \(\frac{4}{11}\)

9. \(0.7083\)

10. \(3.81\)

11. \(0.057\)

12. \(\frac{47}{50}\)

13. \(\frac{23}{50}\)

Page 82

14. \(33 \frac{1}{3}\)

15. \(30.25\%\)

16. \(64\%\)

17. \(147\)

18. \(96\)

19. \(42\)

20. \(36\)

21. \(<\)

22. \(>\)

23. \(=\)

24. \(\frac{19}{5}\) ft, \(\frac{11}{12}\) ft,

25. \(4.0625\) ft

B: \(29\)
# Chapter 4 Assessment Answer Key

**Page 83, Extended-Response Test**

**Scoring Rubric**

<table>
<thead>
<tr>
<th>Level</th>
<th>Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>The student demonstrates a <strong>thorough understanding</strong> of the mathematics concepts and/or procedures embodied in the task. The student has responded correctly to the task, used mathematically sound procedures, and provided clear and complete explanations and interpretations. The response may contain minor flaws that do not detract from the demonstration of a thorough understanding.</td>
</tr>
<tr>
<td>3</td>
<td>The student demonstrates an <strong>understanding</strong> of the mathematics concepts and/or procedures embodied in the task. The student's response to the task is essentially correct with the mathematical procedures used and the explanations and interpretations provided demonstrating an essential but less than thorough understanding. The response may contain minor errors that reflect inattentive execution of the mathematical procedures or indications of some misunderstanding of the underlying mathematics concepts and/or procedures.</td>
</tr>
<tr>
<td>2</td>
<td>The student has demonstrated only a <strong>partial understanding</strong> of the mathematics concepts and/or procedures embodied in the task. Although the student may have used the correct approach to obtaining a solution or may have provided a correct solution, the student's work lacks an essential understanding of the underlying mathematical concepts. The response contains errors related to misunderstanding important aspects of the task, misuse of mathematical procedures, or faulty interpretations of results.</td>
</tr>
<tr>
<td>1</td>
<td>The student has demonstrated a <strong>very limited understanding</strong> of the mathematics concepts and/or procedures embodied in the task. The student's response to the task is incomplete and exhibits many flaws. Although the student has addressed some of the conditions of the task, the student reached an inadequate conclusion and/or provided reasoning that was faulty or incomplete. The response exhibits many errors or may be incomplete.</td>
</tr>
<tr>
<td>0</td>
<td>The student has provided a <strong>completely incorrect</strong> solution or uninterpretable response, or no response at all.</td>
</tr>
</tbody>
</table>


Chapter 4 Assessment Answer Key

Page 83, Extended Response Test
Sample Answers

In addition to the scoring rubric found on page A35, the following sample answers may be used as guidance in evaluating extended response assessment items.

1. a. Yes, because 30 and 24 are both divisible by 3.

   b. List the factors of each number.
   
   factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
   factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
   
   Identify the common factors.
   
   common factors: 1, 2, 3, and 6

2. a. Julian’s batting average: \( \frac{27}{72} \), 0.375, 37.5%

   b. Write a fraction with the number of games won over the number of games pitched.
   
   I would express this fraction as a decimal. \( \frac{125}{200} = 0.625 \), 62.5%

   c. 6 ft; it is called the greatest common factor.
   
   Multiples of 25: 25, 50, 75, 100, 125, 150, 175.
   
   Multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120, 135, 150
   
   The LCM is 75.

   d. Five feet; 5 is a factor of 25 and 15.
   
   It is also close to 6 feet which is the spacing used across the front.
Chapter 4 Assessment Answer Key

Standardized Test Practice
Page 84

1. ○ ○ ● ○

2. ○ ○ ○ ●

3. ○ ○ ● ○

4. ○ ● ○ ○

5. ● ○ ○ ○

6. ○ ● ○ ○

7. ○ ○ ● ○

8. ● ○ ○ ○

9. ● ○ ○ ○

Page 85

10. ○ ○ ● ○

11. ○ ○ ○ ●

12. ● ○ ○ ○

13. ● ○ ○ ○

14. ○ ○ ○ ●

15. ○ ● ○ ○

16. ○ ○ ○ ●

17. ○ ● ○ ○
Chapter 4 Assessment Answer Key

Standardized Test Practice
Page 86

18. _______ 108 _______

19. _______ 18 _______

20. _______ 167° F _______

21. _______ −0.3 _______

22. _______ 5 \times 7 \times a \times a \times b \times b _______

23. _______ 7 _______

25. a. \( \frac{9}{10} \), 0.99, 99.99%, \( \frac{100}{99} \)

b. 99.99%; Sample answer: The difference between 99.99% = 0.99999 and 1 is the least.